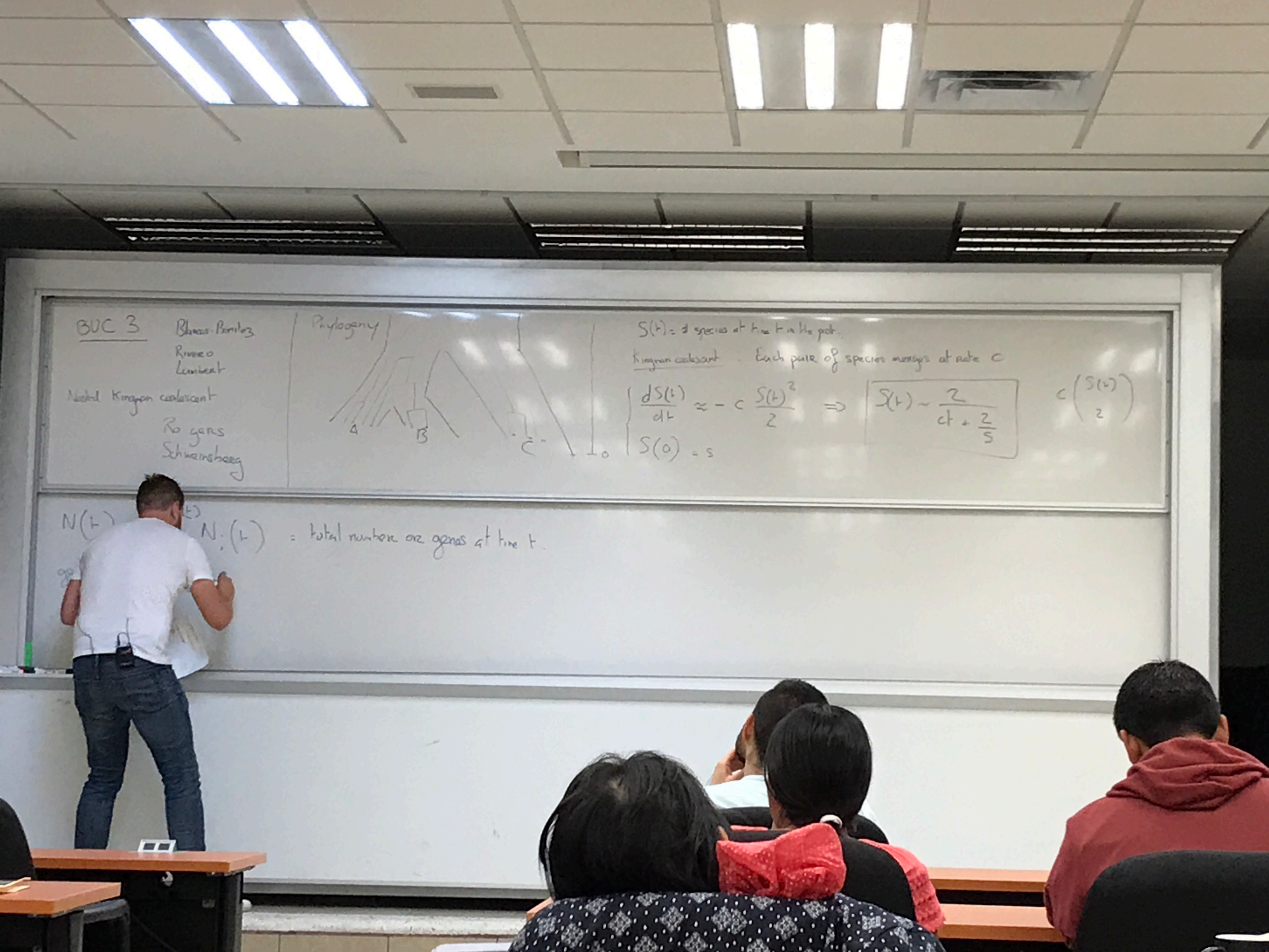

BUC 3
Blanco-Benitez
Rivero
Lambert
Nested Kingman coalescent
Rogers
Schweinsberg
Phylogeny
A
B
C
0
S(t) = # species at time t in the past.
Kingman coalescent . Each pair of species merges at rate c
$\frac{dS(t)}{dt} \approx -c \frac{S(t)^2}{2}$
$S(0) = s$
$\Rightarrow \boxed{S(t) \sim \frac{2}{ct + \frac{2}{s}}}$
$c\binom{S(t)}{2}$
N(t)
$N_i(t)$ = total number of genes at time t.

BUC 3
Phylogeny
A
B
C
S(t) = # species at time t in the past.
Kingman coalescent : Each pair of species merges at rate c
dS(t)/dt ≈ − c S(t)^2/2 ⟹ S(t) ~ 2/(ct + 2/s)
S(0) = s
c (S(t) choose 2)
N(t) = Σ_{i=1}^{S(t)} N_i(t) : total number of genes at time t.
N_i(0) = n >> s ⟹ the first events are
N_i(t) ≈ 2/(t + 2/n) ≈ 2/t (if t >> 1/n
N(t) ≈ 2s/t

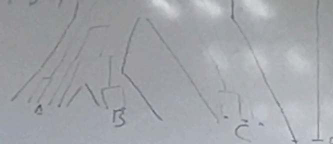

If t >> 1/s ; Coalescence of
genes AND species -
Species coalescence => Ni(t)
"double" its size
we cannot approximate it by
its expectation
Phylogeny
A
B
C
=> Ni(t) ≈ W/t
W is a random variable
LLN => N(t) ~ S(t) . E(W)/t
~ 2E(W)/ct²
cies merges at rate c
S(t) ~ 2/(ct + 2/s)
log N(t)
t
1/s

If $t \gg \frac{1}{s}$. Coalescence of genes AND species –

Species coalescence $\Rightarrow N_i(t)$ "doubles" its size

we cannot approximate it by its expectation

Phylogeny



$\Rightarrow N_i(t) \approx \frac{W}{t}$

W is a random variable

LLN $\rightarrow N(t) \sim S(t) \cdot \frac{E(W)}{t} \sim \boxed{\frac{2E(W)}{ct^2}}$

...cies merges at rate c

$$\boxed{S(t) \sim \frac{2}{ct + \frac{2}{s}}}$$

$$c\binom{S(t)}{2}$$

Obs. Lambert, Schertzer $t \sim \frac{1}{s}$

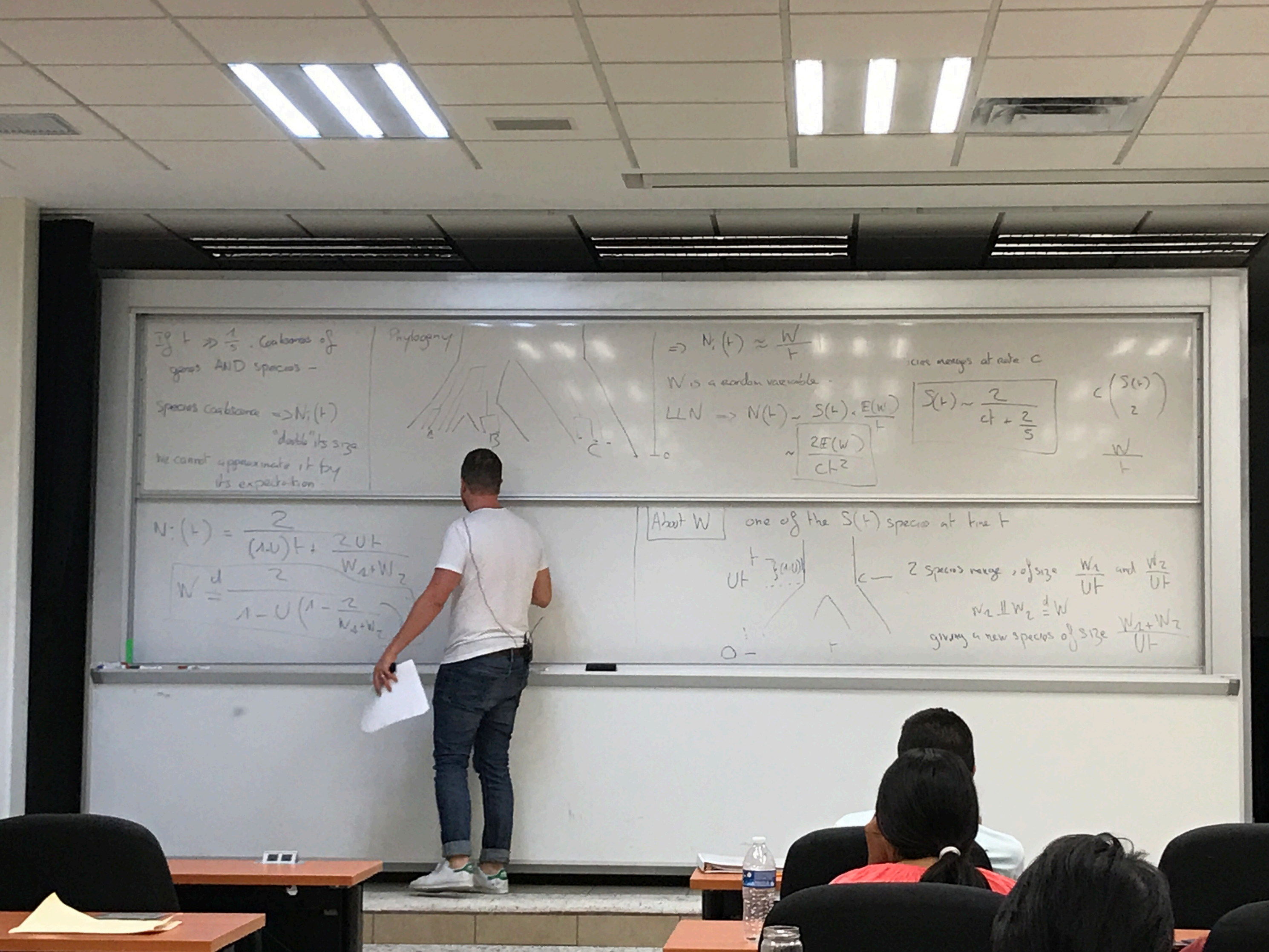
Smoluchowski equation with mass loss

$P_t(x)$ = # of species with mass x

x y

$x+y$

About W

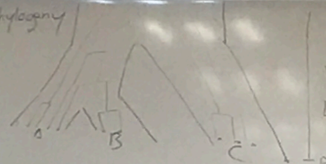

If $t \gg \frac{1}{s}$. Coalescence of genes AND species -
Species coalescence $\Rightarrow N_i(t)$ "double" its size
we cannot approximate it by its expectation
Phylogeny
A
B
C
0
$\Rightarrow N_i(t) \approx \frac{W}{t}$
W is a random variable.
LLN $\rightarrow N(t) \sim S(t) \cdot \frac{E(W)}{t} \sim \boxed{\frac{2E(W)}{ct^2}}$
species merge at rate c
$\boxed{S(t) \sim \frac{2}{ct + \frac{2}{s}}}$
$c\binom{S(t)}{2}$
$\frac{W}{t}$
$N_i(t) = \frac{2}{(1-U)t + \frac{2Ut}{W_1+W_2}}$
$\boxed{W \overset{d}{=} \frac{2}{1-U\left(1-\frac{2}{W_1+W_2}\right)}}$
About W
one of the S(t) species at time t
t
Ut
(1-U)t
c
0
t
2 species merge, of size $\frac{W_1}{Ut}$ and $\frac{W_2}{Ut}$
$W_1 \perp\!\!\!\perp W_2 \overset{d}{=} W$
giving a new species of size $\frac{W_1+W_2}{Ut}$

If $t \gg \frac{1}{s}$ Coalesces of genes AND species –

Species coalescence $\Rightarrow N_i(t)$ "double" its size

we cannot approximate it by its expectation

Phylogeny



$\Rightarrow N_i(t) \simeq \frac{W}{t}$

W is a random variable

LLN $\to N(t) \sim S(t) \cdot \frac{E(W)}{t} \sim \boxed{\frac{2\mathbb{E}(W)}{ct^2}}$

…cies merges at rate c

$$\boxed{S(t) \sim \frac{2}{ct + \frac{2}{s}}}$$

$c\binom{S(t)}{2}$

$\frac{W}{t}$

$$N_i(t) = \frac{2}{(1-U)t + \frac{2Ut}{W_1+W_2}}$$

$$\boxed{W \overset{d}{=} \frac{2}{1 - U\left(1 - \frac{2}{W_1+W_2}\right)}}$$

$\boxed{\text{About } U}$

Yule process : U uniform

Berestycki $\times 2$

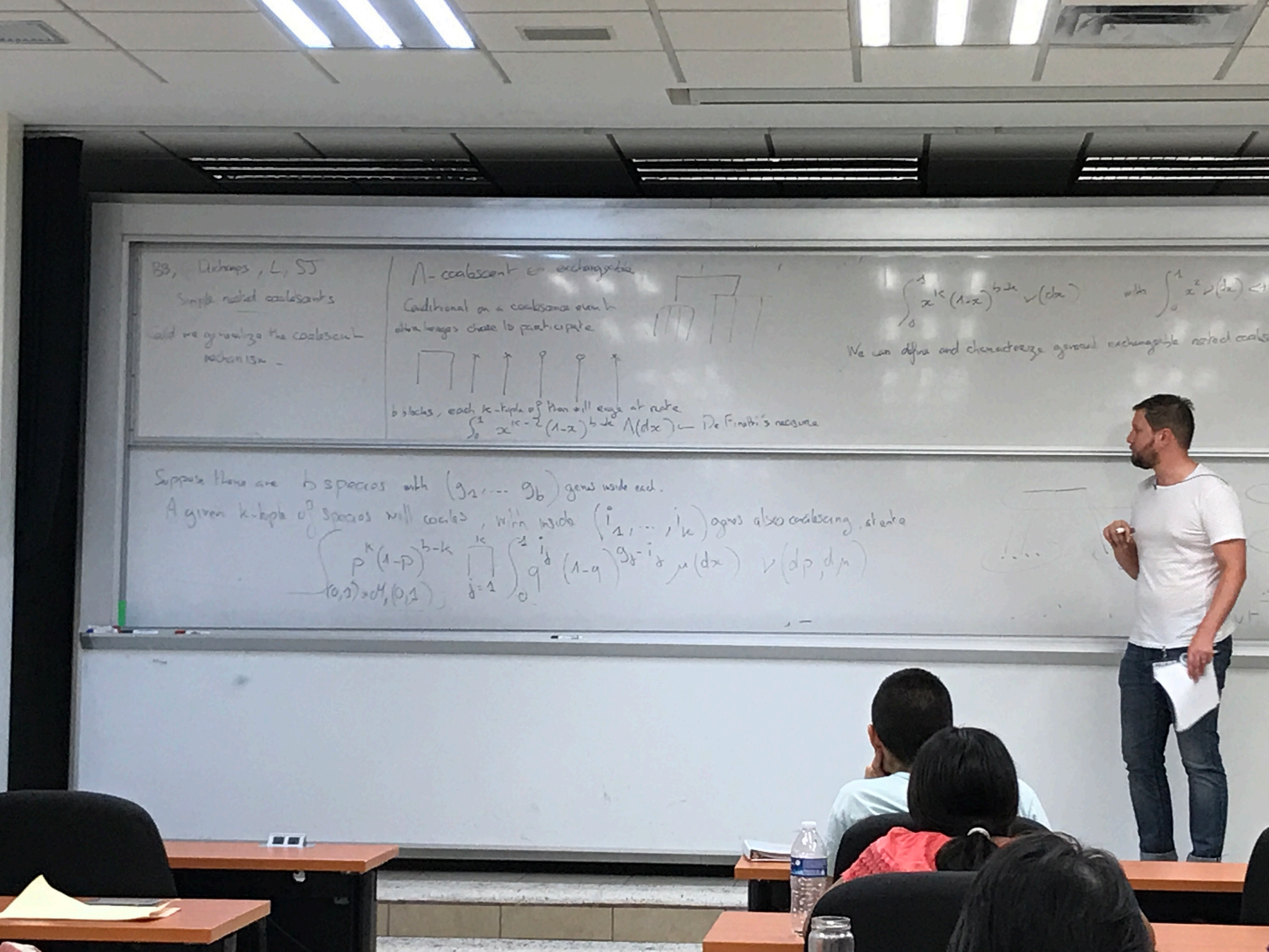
Reverse Kingman is a time changed Yule process

this time change does not affect so much

<u>Teo</u> Suppose $\frac{1}{s_j} \ll t_j \ll 1$ and $\frac{1}{\sqrt{n_j s_j}} \ll t_j$

$$t_j^2 N(t_j) \xrightarrow[j\to\infty]{\mathbb{P}} \frac{2\mathbb{E}(W)}{c}$$


BB, Duchamps, L, SJ
Simple nested coalescents
Could we generalize the coalescent mechanism
Λ-coalescent ↔ exchangeable
Conditional on a coalescence event
b blocks, each k-tuple of them will merge at rate
∫₀¹ x^{k-2} (1-x)^{b-k} Λ(dx) ← De Finetti's measure
We can define and characterize general exchangeable nested coalescents


BB, Duchamps, L, SJ
Simple nested coalescents
Could we generalize the coalescent mechanism
Λ-coalescent ⟺ exchangeable
Conditional on a coalescence event
other lineages choose to participate
b blocks, each k-tuple of them will merge at rate
$\int_0^1 x^{k-2}(1-x)^{b-k}\,\Lambda(dx)$ ← De Finetti's measure
$\int_0^1 x^k(1-x)^{b-k}\,\nu(dx)$ with $\int_0^1 x^2\,\nu(dx)$
We can define and characterize general exchangeable nested coalescents
Suppose there are b species with $(g_1, \dots g_b)$ genes inside each.
A given k-tuple of species will coalesce, with inside $(i_1, \dots, i_k)$ genes also coalescing, at rate
$\int_{(0,1)\times\mathcal{M}_1(0,1)} p^k(1-p)^{b-k} \prod_{j=1}^{k} \int_0^1 q^{i_j}(1-q)^{g_j-i_j}\,\mu(dx)\;\nu(dp,d\mu)$

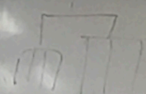
BB, Duchamps, L, SJ

Simple nested coalescents

could we generalize the coalescent mechanism.

Λ-coalescent $\leftrightarrow$ exchangeable

Conditional on a coalescence event, lineages choose to participate



b blocks, each k-tuple of them will merge at rate

$\int_0^1 x^{k-2}(1-x)^{b-k}\Lambda(dx)$ — De Finetti's measure

$\int_0^1 x^{k}(1-x)^{b-k}\nu(dx)$ with $\int_0^1 x^2\nu(dx) < +\infty$

We can define and characterize general exchangeable nested coalescents

Suppose there are b species with $(g_1, \dots g_b)$ genes inside each.

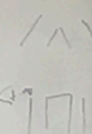
A given k-tuple of species will coalesce, with inside $(i_1, \dots, i_k)$ genes also coalescing at rate

$$\int_{(0,1)\times M_1(0,1)} p^k(1-p)^{b-k}\prod_{j=1}^{k}\int_0^1 q^{i_j}(1-q)^{g_j-i_j}\mu(dx)\,\nu(dp,d\mu)$$

$\int \nu(dp,d\mu)\,p^2 < +\infty$

and

$\int \nu(dp,d\mu)\,p\int_0^1 q^2\mu(dq) < +\infty$



Ex $\nu'(dp,dm,db)\,\delta_{\mu_{a,b}}(d\mu)$