

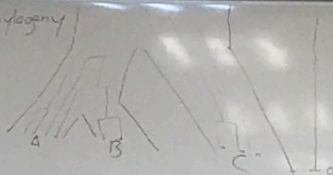
BUC 3

Blass Pointes
Riviera
Lumbert

Nostal Kingdom caducant

No gens
Schwarzhof

Phylogeny



$S(t)$ = # species at time t in the past.

Kingdom caducant - Each pair of species merges at rate c

$$\left\{ \begin{aligned} \frac{dS(t)}{dt} &\approx -c \frac{S(t)^2}{2} \\ S(0) &= s \end{aligned} \right. \Rightarrow$$

$$S(t) = \frac{2}{ct + \frac{2}{s}}$$

$$c \binom{S(t)}{2}$$

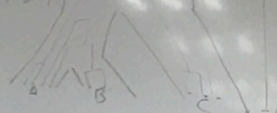
$N(t)$ = total number of genes at time t .



BUC 3

Blauer Fleck
Kranz
Leinwand
Viel Kragen
kein Kragen
Schwanz

Phylogeny



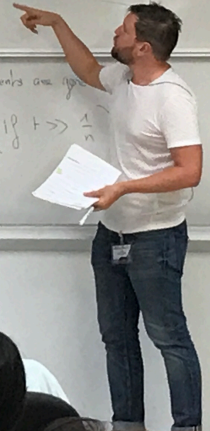
$S(t)$ = # species at time t in the past.
Kingman coalescent: Each pair of species merges at rate c

$$\frac{dS(t)}{dt} \approx -c \frac{S(t)^2}{2}$$

$$S(0) = s$$

$$S(t) \approx \frac{2}{ct + \frac{2}{s}}$$

$$c \binom{S(t)}{2}$$



$N(t) = \sum_{i=0}^{2t} N_i(t)$ = total number of genes at time t .
genes coalesce (inside their species) according to a Kingman coalescent

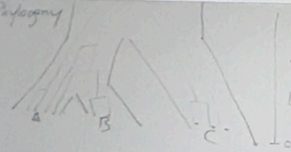
$$N_i(0) = n \gg s \Rightarrow \text{the first events are } \text{ignorable}$$

$$N_i(t) \approx \frac{2}{t + \frac{2}{n}} \approx \frac{2}{t} \quad (10 \gg 1 \Rightarrow \frac{1}{n})$$

$$N(t) \approx \frac{2s}{t}$$

$\frac{1}{2}t \Rightarrow \frac{1}{2}$, Combos of
 genes AND species -
 Species combine $\Rightarrow N(t)$
 "double" its size
 "no comb" approximated by
 its expectation

Phylogeny



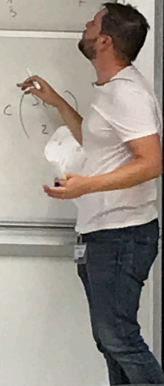
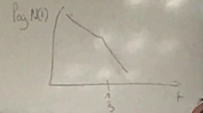
$$\Rightarrow N(t) \approx \frac{W}{T}$$

W is a random variable

$$\text{LLN} \rightarrow N(t) \sim \frac{S(t) \cdot E(W)}{T} \\
 \sim \frac{2E(W)}{ct^2}$$

genes merge at rate c

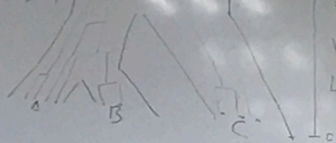
$$S(t) \sim \frac{2}{ct + \frac{2}{S}}$$



if $t \gg \frac{1}{s}$, number of
 spec. AND species -

Species coexistence $\Rightarrow N_s(t)$
 "double" its size
 we cannot approximate it by
 its expectation

Phylogeny



$$\Rightarrow N_s(t) \approx \frac{W}{F}$$

W is a random variable

$$\text{LLN} \rightarrow N(t) \sim \frac{S(t) \cdot E(W)}{F}$$

$$\sim \frac{2E(W)}{cF^2}$$

times merge's at rate c

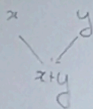
$$S(t) \sim \frac{2}{ct + \frac{2}{s}}$$

$$c \binom{S(t)}{2}$$

Obs: Lambert, Schweitzer $t \sim \frac{1}{s}$

Stochastic equation with mass loss

$$P_s(x) = \# \text{ of spec. with mass } x$$

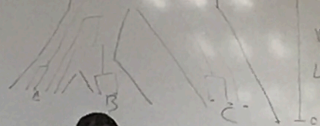


About W

If $t \gg \frac{1}{c}$, Coalesces of
genes AND species -

Species coalesce $\Rightarrow N_s(t)$
"doubt" its size
we cannot approximate it by
its expectation.

Phylogeny



$$\Rightarrow N_s(t) \approx \frac{W}{t}$$

W is a random variable.

$$\text{LLN} \Rightarrow N_s(t) \sim \frac{S(t) \cdot E(W)}{t} \\ \sim \frac{2E(W)}{ct^2}$$

cross merges at rate c

$$S(t) \sim \frac{2}{ct + \frac{2}{5}}$$

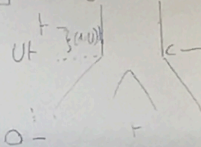
$$c \binom{S(t)}{2}$$

$$\frac{W}{t}$$

$$N_s(t) = \frac{2}{(1W)t + \frac{2Ut}{W_1 + W_2}}$$

$$W = \frac{d}{1 - U \left(1 - \frac{2}{W_1 + W_2} \right)}$$

About W one of the $S(t)$ species at time t



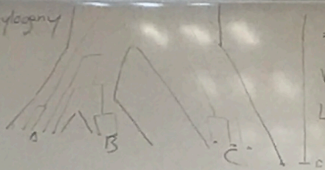
2 species merge, of size $\frac{W_1}{Ut}$ and $\frac{W_2}{Ut}$

$W_1 \cup W_2 = W$
giving a new species of size $\frac{W_1 + W_2}{Ut}$

$\frac{1}{2} t \gg \frac{1}{S}$. Coalesces of
genes AND species -

Species Coalescence $\Rightarrow N_i(t)$
"dabb" its size
we cannot approximate it by
its expectation

Phylogeny



$$\Rightarrow N_i(t) \approx \frac{W}{t}$$

W is a random variable

$$\text{LLN} \rightarrow N(t) \sim \frac{S(t) \cdot E(W)}{t} \\ \sim \frac{2E(W)}{ct^2}$$

series averages at rate c

$$S(t) \sim \frac{2}{ct + \frac{2}{S}}$$

$$c \binom{S(t)}{2} \\ \frac{W}{t}$$

$$N_i(t) = \frac{2}{(1-U)t + \frac{2Ut}{W_1 + W_2}}$$

$$W = \frac{d}{2} \frac{1 - U \left(1 - \frac{2}{W_1 + W_2}\right)}{1 - U \left(1 - \frac{2}{W_1 + W_2}\right)}$$

About U

Y_{obs} process: U uniform

Beneschki ≈ 2

Reverse Kinship is a
time changed Y_{obs} process

this time change does not affect so much

Theo

Suppose

$$\frac{1}{S_d} \ll t_j \ll 1 \text{ and } \frac{1}{W_j S_j} \ll t_j$$

$$\frac{1}{2} N(t_j) \xrightarrow{d \rightarrow \infty} \frac{2E(W)}{c}$$

$$\frac{1}{2}$$

BB, Wickens, L, SJ

Simple nested coalitions

Could we generalize the coalitional mechanism -

Λ -coalition \leftrightarrow exchangeable

Conditional on a coalition event ω , all strategies chosen to participate



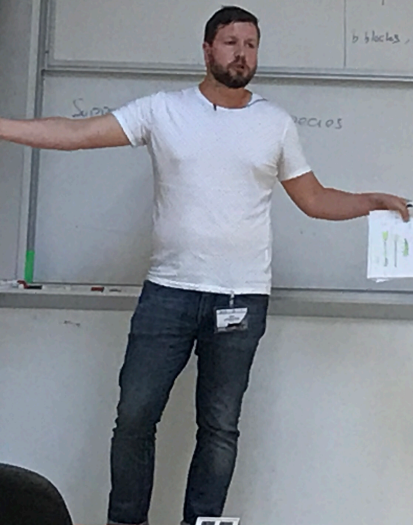
b blocks, each k -tuple of them will equal at center

$$\int_0^1 x^{k-1} (1-x)^{b-k} \Lambda(dx) \leftarrow \text{De Finetti's measure}$$

$$\int_0^1 x^k (1-x)^{b-k} \nu(dx) \quad \text{with} \quad \int_0^1 x^k \nu(dx) = 0$$

We can define and characterize general exchangeable coalitions

Suppose ν is a measure



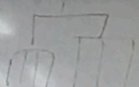
BB, Dirichlet, L, SJ

Simple nested coalescents

could we generalize the coalescent mechanism -

Λ -coalescent or exchangeable

Conditional on a coalescence event k lineages chose to participate



b blocks, each k -tuple of them will merge at rate $\int_0^1 x^{k-2} (1-x)^{b-k} \Lambda(dx)$ De Finetti's measure

$$\int_0^1 x^{k-2} (1-x)^{b-k} \nu(dx)$$

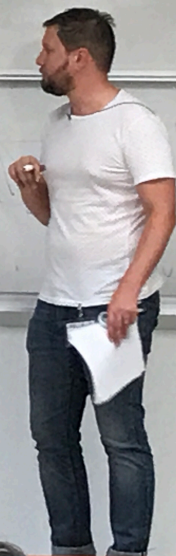
$$\text{with } \int_0^1 x^2 \nu(dx) < 1$$

We can define and characterize general exchangeable nested coalescents

Suppose there are b species with (g_1, \dots, g_b) genes inside each.

A given k -tuple of species will coalesce, with inside (i_1, \dots, i_k) genes also coalescing, at rate

$$\int_0^1 p^{k-2} (1-p)^{b-k} \prod_{j=1}^k q_j^{i_j} (1-q_j)^{g_j-i_j} \mu(dx) \nu(dp, dq)$$



BB, Uehlings, L, SS

Simple nested coalescents

could we generalize the coalescent mechanism -

Λ -coalescent \Leftarrow exchangeable

Conditional on a coalescence event k other lineages choose to participate



b blocks, each k -tuple of them will merge at once
 $\int_0^1 x^{k-2} (1-x)^{b-k} \Lambda(dx) \leftarrow$ De Finetti's measure

$$\int_0^1 x^{k-1} (1-x)^{b-k} \nu(dx) \quad \text{with} \quad \int_0^1 x^2 \nu(dx) < \infty$$

We can define and characterize general exchangeable nested coalescents

Suppose there are b species with (g_1, \dots, g_b) genes inside each.

A given k -tuple of species will coalesce, with inside (i_1, \dots, i_k) genes also coalescing at once

$$\int_{(0,1)^k} p^k (1-p)^{b-k} \prod_{i=1}^k q^{i_i} (1-q)^{g_i - i_i} \mu(dx) \nu(dp, d\mu)$$

$$\int \nu(dp, d\mu) p^2 \rightarrow +\infty \quad \wedge$$

$$\int \nu(dp, d\mu) P \int_0^1 q^2 \mu(dq) \rightarrow \infty \quad \wedge$$

$$E \nu(dp, d\mu, d\mu) \partial_{\mu}^2 \nu(dp)$$



