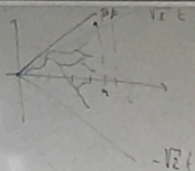




(2_n) lifetime
 (β_n) displacement
 D_n birth time; $X_n(t)$ location at time t



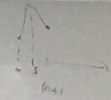
$\limsup_{t \rightarrow \infty} \frac{1}{t} \max_{x \in Z_t} X_n(t) \rightarrow \sqrt{2}$ as
 $t \rightarrow \infty$ as
 1) Upper bound
 Take $\beta > \sqrt{2}$ & $\left[\sum_{x \in Z_t} \mathbb{1}_{\{X_n(t) > \beta t\}} \right]$
 $\rightarrow \forall$ integers n by comp, $\max_{x \in Z_n} X_n(t) < \beta n$

discussion
 to measure
 "fact"



Then show that Y is infinite large enough.

$$\sum_{n \in \mathbb{Z}_m} \frac{1}{2^{|n|}} \left\{ \max_{S \in \mathcal{S}(n)} |X_{n,S}(t) - X_{n,S}(t)| \geq \frac{1}{m^{\beta}} \right\} = 0 \quad \forall m \text{ large enough}$$

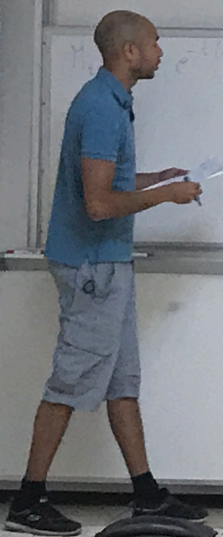


Let β be small $0 < \beta < 1/2$

$$\mathbb{E} \left[\sum_{n \in \mathbb{Z}_m} \frac{1}{2^{|n|}} |Y_{n,S}(t) - Y_{n,S}(t)| \right]$$

Take t_0 big enough. Construct a set S around n of size $|S| = k > 0$.

$$Y_{n,S}(t) = e^{-t(1+\frac{\beta}{2})} \sum_{n \in \mathbb{Z}_m} e^{\frac{\beta}{2}|n|} Y_{n,S}(t)$$



$$P^* = \frac{1}{2} + (1 - \frac{\beta}{2}) \sum_{x \in \mathbb{Z}^d} \beta^{|x|} \delta_x$$

Under the measure P^* , the process is a branching process with offspring distribution β .

We define P^* probability measure on the space \mathcal{R} under P^* , (W, \mathcal{R}) with the following distribution \mathcal{R} is a uniform tree in the set of rays of the binary tree.



Let $\beta < 2$.

$$\mathbb{E} \left[\sum_{x \in \mathbb{Z}^d} \frac{1}{|x|} \mathbb{1}_{\|x\| \leq \beta \|x\|} \right]$$

Take β big enough, then the above is > 1 .

grows as $\beta \rightarrow 2^-$

at line $\beta = 2$.

$$e^{-\lambda(1+\frac{\sigma^2}{2})} \dots$$

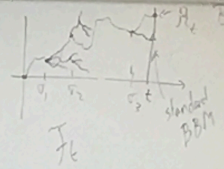
indicates a change of probability that we describe

consider the space of marked trees with associated root ω

We define P^* probability measure on this space under P^* , (ω, R) with the following distribution R is a uniform law on the set of rays of the binary tree.



Missing!



$B_s + Rt$
(σ_k) PPP (?)

Proposition: Recall U_t killing operator at time t on the space of marked trees.

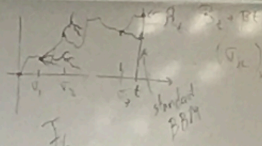
$$E^*[\phi(U_t(\omega), R_t)] = e^{-t(1+\frac{\sigma^2}{2})} E\left[\sum_{x \in \mathcal{L}_t} \phi(x) \otimes (U_t(\omega), s)\right]$$



Proof. Take x_0 in the binary tree, at generation k

$$E \left[\mathbb{1}_{R_t=x} \prod_{s=0}^{t-1} F_s \right] = e^{-t(H_0 \frac{\sigma^2}{2})} E \left[\mathbb{1}_{x \in Z_t} e^{\sum_{s=0}^{t-1} \mathbb{1}_{x \in Z_s}} \prod_{s=0}^{t-1} F_s \right]$$

where (i) $F_s = F_2(\lambda_s, x \in Z_s)$
 (ii) $F_s = F_2(X_s(s), s \leq t)$
 F_s is a measurable function of (λ_s, S_s) for y is not an ancestor of x_0

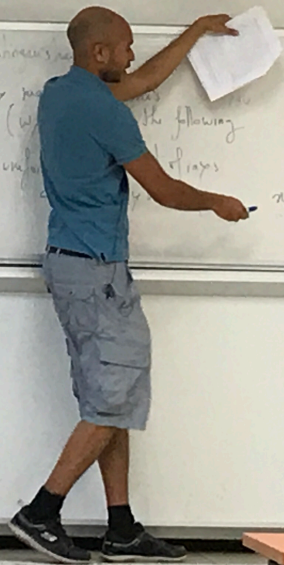


Proposition. Recall U_t falling number at time t on the space of marked trees.

$$E \left[\phi(U_t(\omega), R_t) \right] = e^{-t(H_0 \frac{\sigma^2}{2})} E \left[\sum_{x \in Z_t} e^{\sum_{s=0}^{t-1} \mathbb{1}_{x \in Z_s}} \phi(U_t(\omega), x) \right]$$

$M_t(x) = e^{-t(H_0 \frac{\sigma^2}{2})} \sum_{x \in Z_t} e^{\sum_{s=0}^{t-1} \mathbb{1}_{x \in Z_s}}$
 We obtain a family of probabilities that are decreasing over time.

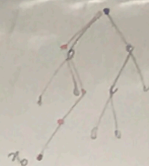
We consider the space of marked trees with distances...
 We define P^* probability measure on the space. Under P^* , (ω, R) has the following distribution. R is a variable of rays.



Left hand side: $E[\mathbb{1}_{X_t = x_0} F_1 F_2 F_3] = 2^{-k} E[\mathbb{1}_{\tilde{X}_0 \leq t < \tilde{X}_1, \tilde{X}_2} F_1(\tilde{X}_2, x < x_0)] E[F_2(B_{s_1}, \beta_s, s \leq t)] E[F_3]$

Probability that $x_0 \in \mathcal{R}$ ($\tilde{X}_x \rightarrow \text{Exp}(z)$)

↑ standard BM



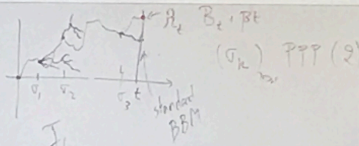
Let x_0 in the binary tree, at generation k

$$E[\mathbb{1}_{R_t = x_0} F_1 F_2 F_3] = e^{-t(1 + \frac{\beta^2}{2})} E\left[\mathbb{1}_{x \in \mathcal{R}_t} e^{\beta X_x(t)} F_1 F_2 F_3 \right]$$

(i) $F_1 = F_2(\lambda_x, x < x_0)$

(ii) $F_2 = F_2(X_x(s), s \leq t)$

F_3 is a measurable function of (λ_y, β_y) for y not an ancestor of x_0



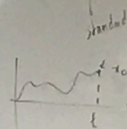
Proportion Recall U_t filling number at time t on the space of marked tree.

$$E[\Phi(U_t(\omega), \mathcal{R}_t)] = e^{-t(1 + \frac{\beta^2}{2})} E\left[\sum_{x \in \mathcal{R}_t} e^{\beta X_x(t)} \Phi(U_t(\omega), x) \right]$$

(1) hand side $E^* [\mathbb{1}_{\lambda_2 < z_0} F_1 F_2 F_3] = S^{-k} E [\mathbb{1}_{\beta_2 s + t < \lambda_2, \lambda_2} F_1(\tilde{\lambda}_2, x < z_0)] [F_2(\beta_2 + \beta_3, s \leq t)] E [F_3]$

It remains to show that

$$S^{-k} E [\mathbb{1}_{\beta_2 s + t < \lambda_2, \lambda_2} F_1(\tilde{\lambda}_2, x < z_0)] = e^{-\lambda_2} E [\mathbb{1}_{x_0 \in z_0} F_1(\lambda_2, x < z_0)]$$



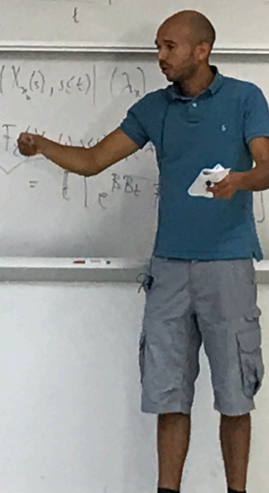
$$E [\mathbb{1}_{x_0 \in z_0} e^{\beta X_0(t)} F_1 F_2 F_3] = E [\mathbb{1}_{x_0 \in z_0} e^{\beta X_0(t)} F_1 F_2] E [F_3]$$

$$= \frac{1}{\lambda_2} E [\mathbb{1}_{x_0 \in z_0} F_1] E [e^{\beta X_0(t)} F_2(\lambda_2, x < z_0)]$$

$$= e^{-\lambda_2} E [F_2(\beta_2 + \beta_3, s \leq t)]$$

or, same as

- where (i) $F_1 = F_1(\lambda_2, x < z_0)$
 (ii) $F_2 = F_2(\lambda_2, x < z_0)$
 (iii) F_3 is a measurable function of (λ_2, β_2) for $y < z_0$



1)1. had side $E[\mathbb{1}_{\lambda_1 < \lambda_2} F_1 F_2 F_3] = e^{-\lambda_1} E[\mathbb{1}_{\lambda_1 < \lambda_2} F_1(\lambda_1, x < \lambda_2)] E[F_2(B_2 + \beta_2, s \leq t)] E[F_3]$

It remains to show that

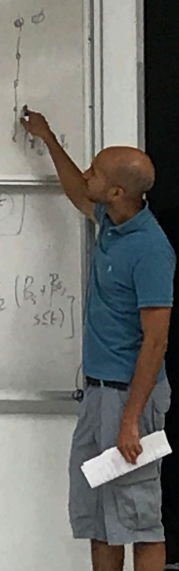
$$e^{-\lambda_1} E[\mathbb{1}_{\lambda_1 < \lambda_2} F_1(\lambda_1, x < \lambda_2)] = e^{-\lambda_1} E[\mathbb{1}_{\lambda_1 < \lambda_2} F_1(\lambda_1, x < \lambda_2)] \Bigg| E[\mathbb{1}_{\lambda_1 < \lambda_2} F_1(\lambda_1, x < \lambda_2)] = E[\mathbb{1}_{b_2 \leq t < b_1 + \lambda_1} F_1(\lambda_1, x < \lambda_2)]$$

$$E[\mathbb{1}_{\lambda_1 < \lambda_2} e^{\beta \lambda_1} F_1 F_2] = E[\mathbb{1}_{\lambda_1 < \lambda_2} e^{\beta \lambda_1} F_2 | E[F_3]]$$

- where:
- (i) $F_1 = F_2(\lambda_1, x < \lambda_2)$
 - (ii) $F_2 = F_2(X_2(t), s \leq t)$
 - (iii) F_3 is a measurable function of (λ_1, λ_2) for y .

$$E[\mathbb{1}_{\lambda_1 < \lambda_2} F_2] e^{\beta \lambda_1} F_2(X_2(t), s \leq t) | (\lambda_1, \lambda_2) = \mathbb{1}_{\lambda_1 < \lambda_2} F_1 E[e^{\beta X_2(t)} F_2(X_2(t), s \leq t) | (\lambda_1, \lambda_2)] = E[e^{\beta B_2} F_2(B_2, s \leq t)] = e^{\beta B_2 - (\frac{\beta^2}{2} t)}$$

Girsanov



left-hand side $E[\mathbb{1}_{\lambda_1 \leq \tau_0} F_1 F_2 F_3] = E[\mathbb{1}_{\lambda_1 \leq \tau_0} F_1(\lambda_1, \tau_0) \mathbb{E}[F_2(B_2 + \beta_0, s \leq t)] E[F_3]]$

It remains to show that

$$E[\mathbb{1}_{\lambda_1 \leq t < \lambda_2} F_1(\lambda_1, \tau_0)] = e^{-t} E[\mathbb{1}_{\lambda_1 \leq t} F_1(\lambda_1, \tau_0)] \cdot E[\mathbb{1}_{\lambda_1 \leq t} F_1(\lambda_1, \tau_0)]$$

$$= E[\mathbb{1}_{\lambda_1 \leq t < \lambda_2 + \lambda_1} F_1(\lambda_1, \tau_0)]$$

$\lambda_1 \leq t < \lambda_2 + \lambda_1 \iff$ PPP of Δ_1 has k points in $[0, t] = e^{-t} \frac{t^k}{k!} E[F_1(\Delta_1)] = e^{-t} \frac{t^k}{k!} E[F_1(\Delta_1)]$

$E[\mathbb{1}_{\lambda_1 \leq t < \lambda_2} F_1(\lambda_1, \tau_0)] = E[F_1(\Delta_1, [0, t])]$

increments of k independent uniform r.v. in $[0, t]$

$$= e^{-t} \frac{t^k}{k!} \cdot \left(e^{-t} \frac{t^k}{k!} E[F_1(\Delta_1, [0, t])] \right)$$

