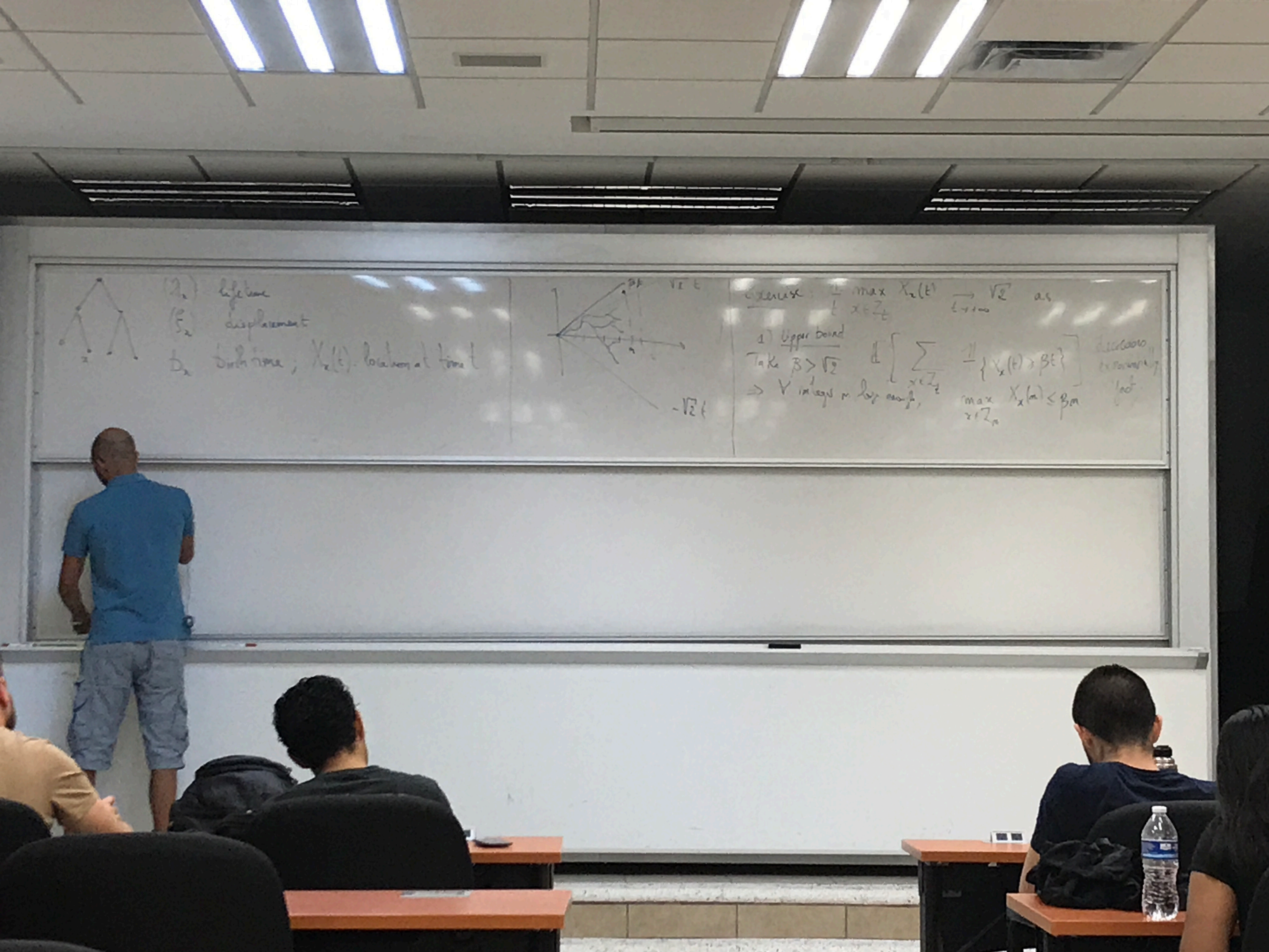
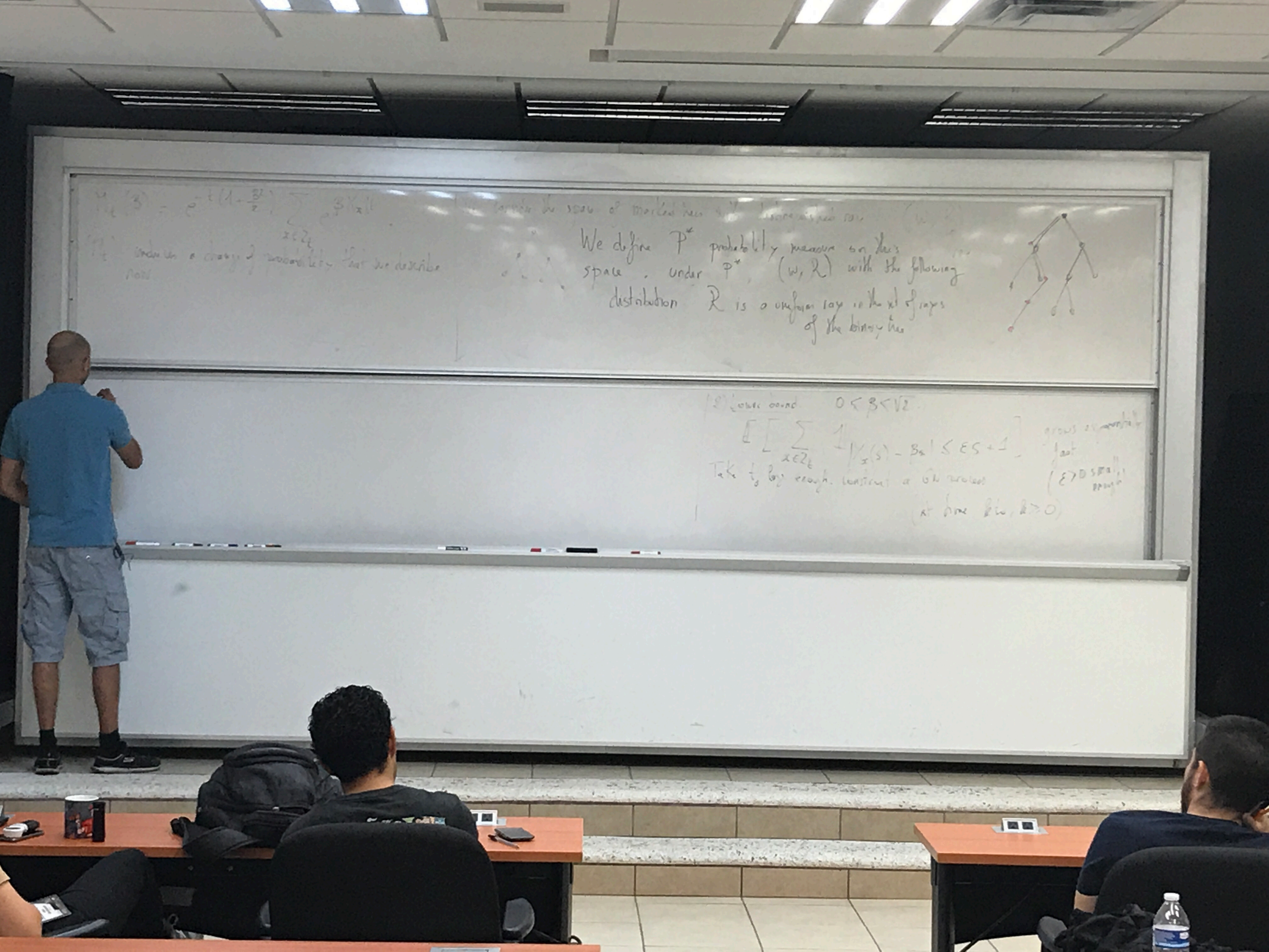
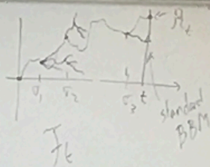

lifetime
displacement
birth time ; X_x(t) location at time t
√2 t
-√2 t
Exercise : 1/t max_{x∈Z_t} X_x(t) → √2 as t→+∞
1) Upper bound
Take β > √2
max_{x∈Z_n} X_x(n) ≤ βn


Thm: Show that ∀ m integer large enough:
= 0 ∀ m large enough
0 < β < √2
Take t large enough. Construct a GW process
(ε > 0 small enough)


We define P* probability measure on this space. under P* (w, R) with the following distribution R is a uniform ray in the set of rays of the binary tree
0 < β < √2
Take t₀ long enough. construct a GW process

We define P^* probability measure on this space. Under P^*, (w, R) with the following distribution: R is a uniform ray in the set of rays of the binary tree

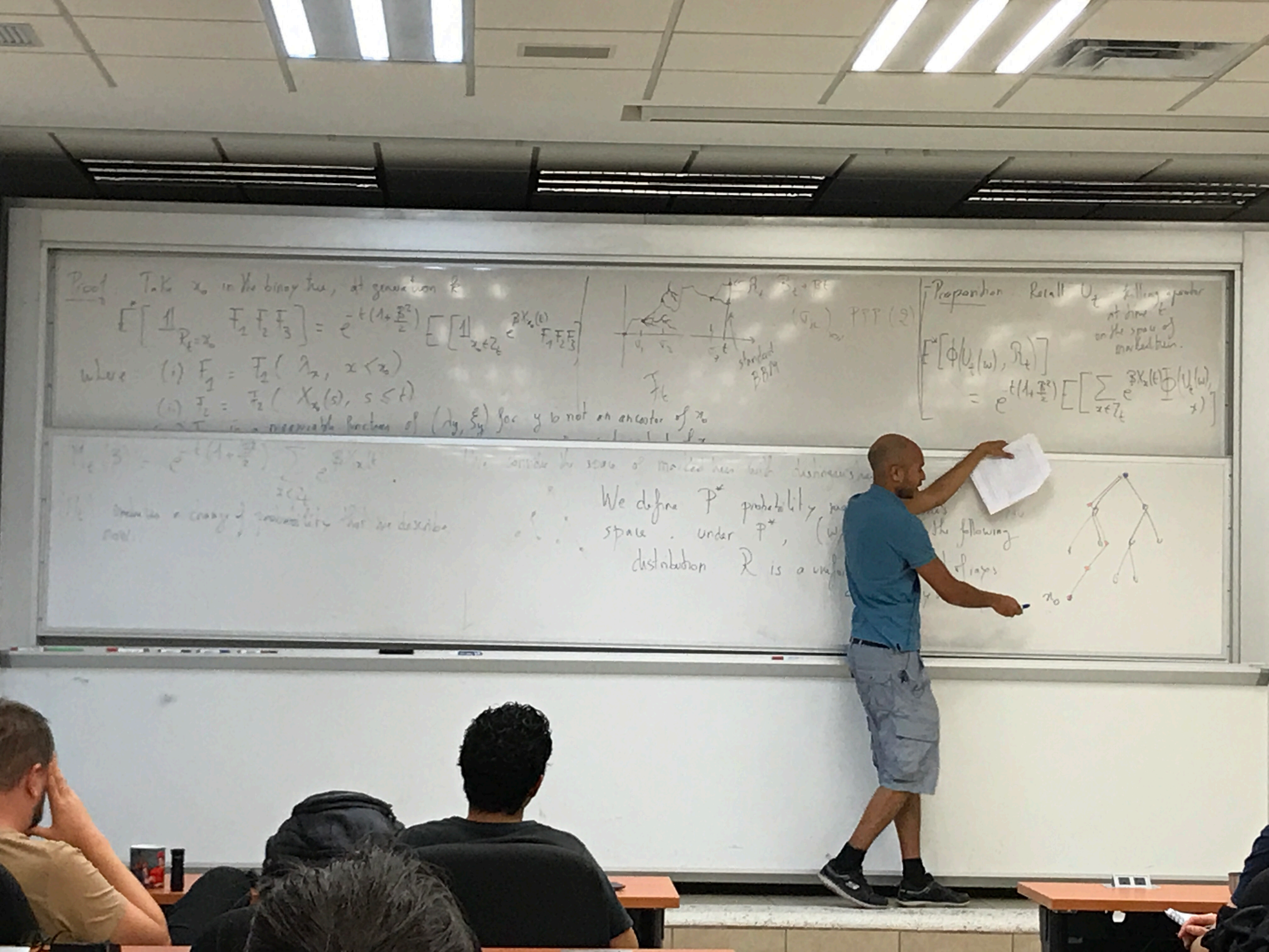
Missing!



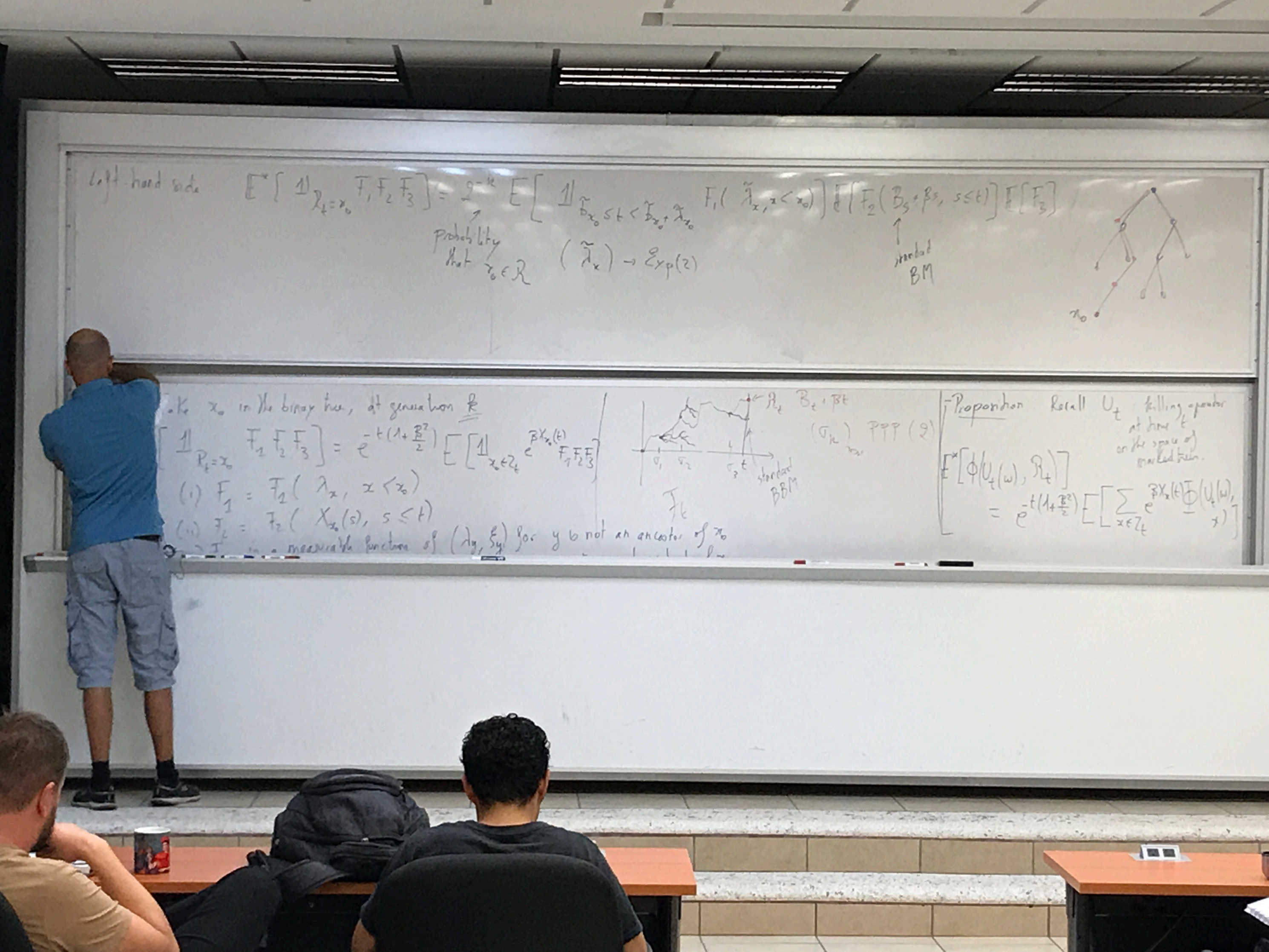
(σ_k) PPP (2)

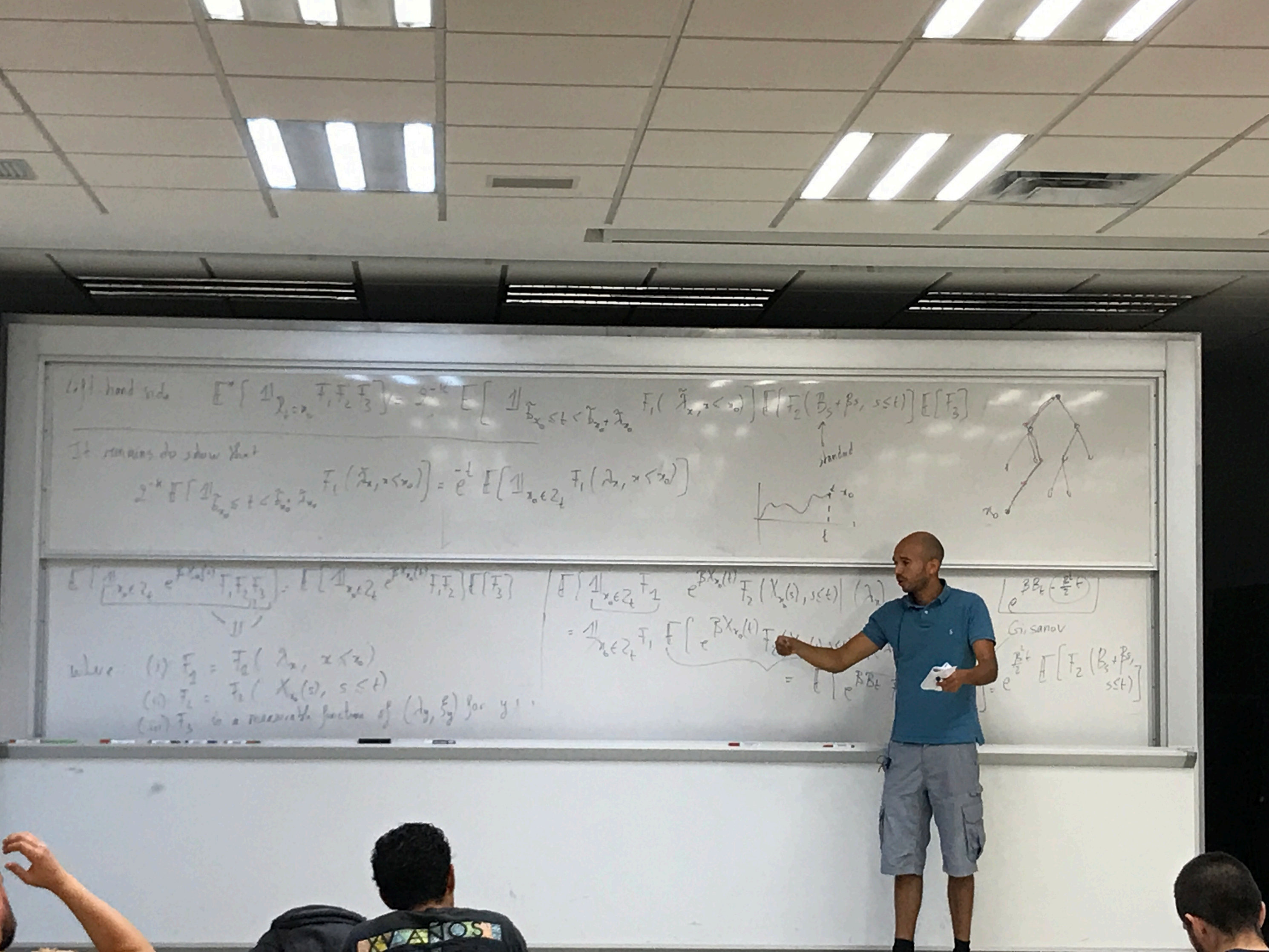
Proposition Recall U_t killing operator at time t on the space of marked trees.

$$E^*\left[\Phi\left(U_t(w), R_t\right)\right] = e^{-t\left(1+\frac{\beta^2}{2}\right)} E\left[\sum_{x \in Z_t} e^{\beta X_x(t)} \Phi\left(U_t(w), x\right)\right]$$



Proof Take x_0 in the binary tree, at generation k
$E^*\left[1_{R_t=x_0} F_1 F_2 F_3\right] = e^{-t(1+\frac{\beta^2}{2})} E\left[1_{x_0 \in Z_t} e^{\beta X_{x_0}(t)} F_1 F_2 F_3\right]$
where (i) $F_1 = F_1(\lambda_x, x < x_0)$
(ii) $F_2 = F_2(X_{x_0}(s), s \le t)$
for y is not an ancestor of x_0
standard BBM
F_t
Proposition Recall U_t killing operator at time t on the space of marked trees.
$E^*[\Phi(U_t(w), R_t)]$
$= e^{-t(1+\frac{\beta^2}{2})} E\left[\sum_{x \in Z_t} e^{\beta X_x(t)} \Phi(U_t(w), x)\right]$
We define P^* probability
space . under P^*,
the following
distribution R is a
x_0


Left-hand side
probability that $x_0 \in \mathcal{R}$
$(\tilde\lambda_x) \to \mathcal{E}xp(2)$
standard BM
x_0
ake x_0 in the binary tree, at generation k
$E[\mathbb{1}_{R_t = x_0} F_1 F_2 F_3] = e^{-t(1+\frac{\beta^2}{2})} E[\mathbb{1}_{x_0 \in Z_t} e^{\beta X_{x_0}(t)} F_1 F_2 F_3]$
(i) $F_1 = F_1(\lambda_x, x < x_0)$
(ii) $F_2 = F_2(X_{x_0}(s), s \le t)$
measurable function of (λ_y, ξ_y) for y not an ancestor of x_0
σ_1 σ_2 σ_3 t
R_t
$B_t + \beta t$
standard BBM
$\mathcal{F}_t$
(σ_k) PPP (2)
Proposition
Recall U_t killing operator at time t on the space of marked trees.
$E^*[\Phi(U_t(\omega), R_t)] = e^{-t(1+\frac{\beta^2}{2})} E[\sum_{x \in Z_t} e^{\beta X_x(t)} \Phi(U_t(\omega), x)]$



Left hand side
It remains to show that
standard
Girsanov
where

Left hand side: $\mathbb{E}\left[\mathbb{1}_{\gamma_t = x_0} F_1 F_2 F_3\right] = 2^{-k}\, \mathbb{E}\left[\mathbb{1}_{\tilde{b}_{x_0} \le t < \tilde{b}_{x_0} + \tilde{\lambda}_{x_0}} F_1(\tilde{\lambda}_x, x < x_0)\right] \mathbb{E}\left[F_2(B_s + \beta s, s \le t)\right] \mathbb{E}[F_3]$

It remains to show that

$$2^{-k}\, \mathbb{E}\left[\mathbb{1}_{\tilde{b}_{x_0} \le t < \tilde{b}_{x_0} + \tilde{\lambda}_{x_0}} F_1(\tilde{\lambda}_x, x < x_0)\right] = e^{-1}\, \mathbb{E}\left[\mathbb{1}_{x_0 \in Z_t} F_1(\lambda_x, x < x_0)\right]$$

$$\mathbb{E}\left[\mathbb{1}_{x_0 \in Z_t} F_1(\lambda_x, x < x_0)\right] = \mathbb{E}\left[\mathbb{1}_{b_{x_0} \le t < b_{x_0} + \lambda_{x_0}} F_1(\lambda_x, x < x_0)\right]$$

$$\mathbb{E}\left[\mathbb{1}_{x_0 \in Z_t} e^{\beta X_{x_0}(t)} F_1 F_2 F_3\right] = \mathbb{E}\left[\mathbb{1}_{x_0 \in Z_t} e^{\beta X_{x_0}(t)} F_1 F_2\right] \mathbb{E}[F_3]$$

where: (i) $F_1 = F_1(\lambda_x, x < x_0)$

(ii) $F_2 = F_2(X_{x_0}(s), s \le t)$

(iii) F_3 is a measurable function of (λ_y, ξ_y) for y ...

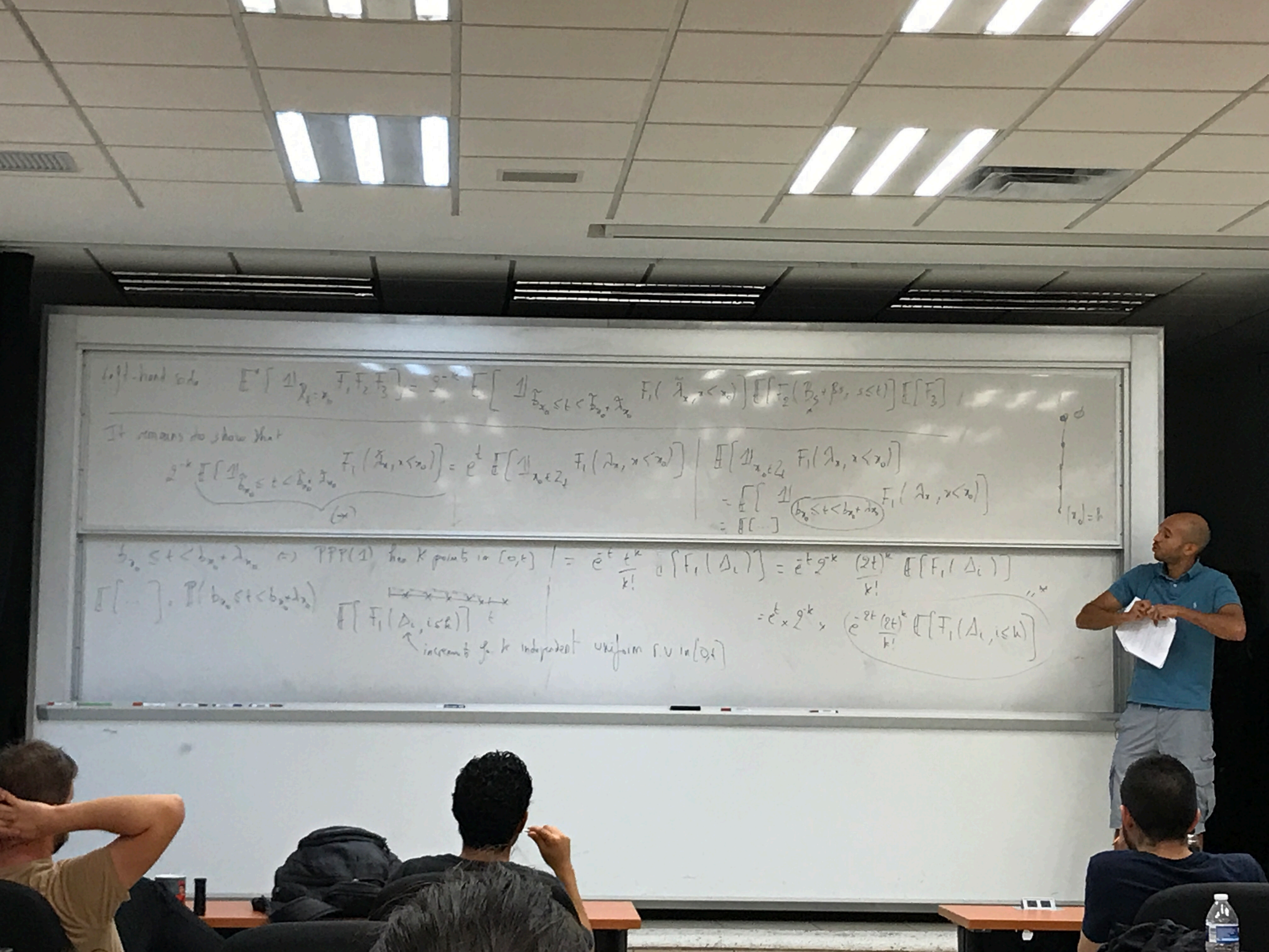
$$\mathbb{E}\left[\mathbb{1}_{x_0 \in Z_t} F_1\, e^{\beta X_{x_0}(t)} F_2(X_{x_0}(s), s \le t) \,\middle|\, (\lambda_x)_x\right]$$

$$= \mathbb{1}_{x_0 \in Z_t} F_1\, \mathbb{E}\left[e^{\beta X_{x_0}(t)} F_2(X_{x_0}(s), s \le t) \,\middle|\, (\lambda_x)_x\right]$$

$$= \mathbb{E}\left[e^{\beta B_t} F_2(B_s, s \le t)\right] = e^{\frac{\beta^2}{2} t}\, \mathbb{E}\left[F_2(B_s + \beta s, s \le t)\right]$$

$e^{\beta B_t - \frac{\beta^2}{2} t}$

Girsanov


left-hand side
It remains to show that
PPP(1) has k points in [0,t]
increments for k independent uniform r.v in [0,t]