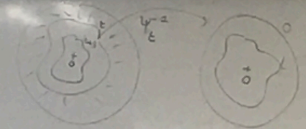
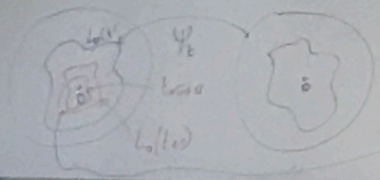
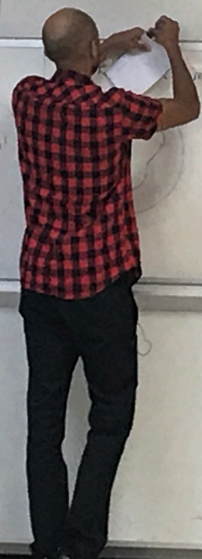
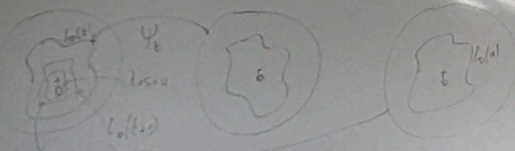


GFF  
 $t > 0$   
 $L_0(t)$ , inside  $L_0(t)$ , GFF + t.  
 $\epsilon > \epsilon$

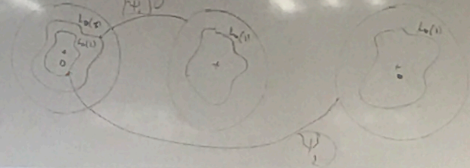


distributed as  $L_0(\epsilon) \perp\!\!\!\perp (L_0(t), r \leq t)$



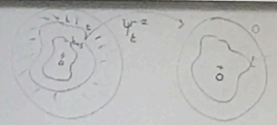


$L_0(\mathbb{R}^d)$   $\mathbb{R}^d$  in layers



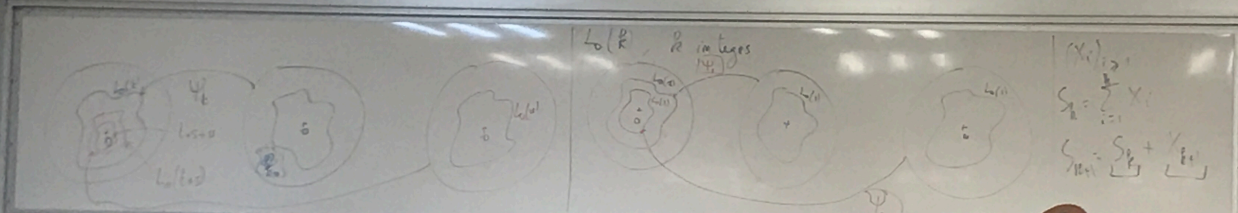
$$\begin{aligned}
 & \{X_i\}_{i=1}^n \\
 & S_T = \sum_{i=1}^n X_i \\
 & S_{T+h} = S_T + \sum_{i=1}^h X_{T+i}
 \end{aligned}$$

GFF  
 $L > 0$   
 $L_0(t)$ , inside  $L_0(t)$ , GFF + t.  
 $t > c$



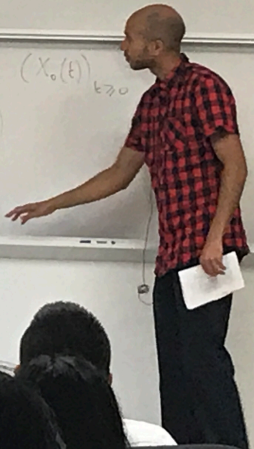
distributed as  $L_0(c) \perp\!\!\!\perp (L_0(\mathbb{R}^d), \mathbb{R}^d)$





$$\begin{aligned}
 (X_i)_{i \geq 1} \\
 S_n &= \sum_{i=1}^n X_i \\
 C_{n+1} &= S_n + \frac{X_{n+1}}{n+1}
 \end{aligned}$$

Exercise: let  $U \subset \mathbb{R}^n$   
 let  $f: U \rightarrow \mathbb{R}^m$ ,  $f(0) = 0$   
 1) Show that  $\dim(L_f(0)) \leq \dim(U)$   
 $\dim(L_f(0)) = \dim(U)$   
 2) Deduce that  $\|X_{t_0}(t) - X_{t_0}(0)\|_{\mathbb{R}^m} \leq \|X_{t_0}(t)\|_{\mathbb{R}^m} \leq \|X_{t_0}(t)\|_{\mathbb{R}^n} \leq \dim(U) \|t - t_0\|$   
 $\leq \dim(U) \|t - t_0\|$

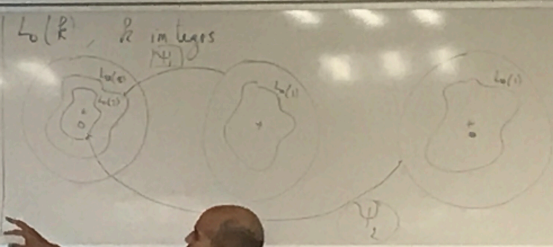


Exercise:

Show that  $e^{2(X_t - X_0)}$  is a martingale

$$e^{2(X_t - X_0)}$$

We admit that  $(X_t)_{t \geq 0}$  is distributed by a BM  $t \geq 0$



$$S_t = \sum_{i=1}^t X_i$$

$$S_{t+1} = S_t + X_{t+1}$$

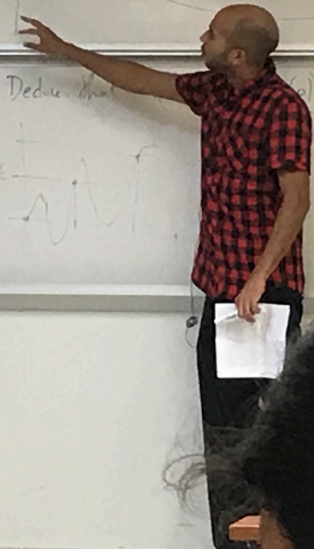
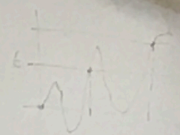
Exercise: let  $Z_t \in \mathbb{R}$

Let  $\mathbb{F}_t = \mathcal{U}_t, \mathcal{U}_0 = \emptyset$

1) Show that  $(L_{\mathbb{F}_t}(t))_{t \geq 0}$  is a martingale

$$X_{\mathbb{F}_t}(t) = -\log \mathbb{P}(L_{\mathbb{F}_t}(t) \leq z)$$

2) Deduce that  $(X_0(t))_{t \geq 0}$  is a martingale





$F = \text{max}$

Show that  $\int_{\mathbb{R}^d} e^{-\beta(x_1^2 + \dots + x_d^2)} dx_1 \dots dx_d > 0$  on  $\mathbb{R}^d$  is a manifold

We admit that  $(X_{t,0})_{t \geq 0}$  is distributed by a BM  $(t \geq 0)$

$$F_t = \sigma(L_2(t)), \beta \in (0, 1) \in \mathbb{R}$$

Question: define  $\frac{dP^\beta}{dP} \Big|_{\mathcal{F}_t} = M_t \leftarrow \text{martingale}$   
Describe  $P^\beta$

$$X_i = \sum_{j=1}^n Y_j$$
  
$$S_{n+1} = S_n + Y_{n+1}$$

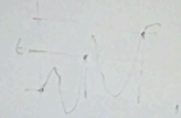
Exercise: let  $Z \in \mathbb{R}^d$

let  $\mathcal{U} \subset \mathbb{R}^d, \mathcal{U} \cap \{Z=0\} = \emptyset$

1) Show that  $\int_{\mathcal{U}} e^{-\beta(L_2(t))} dt > 0 \iff (L_2(t))_{t \geq 0} > 0$

$$X_{2,0}(t) = \log(R(L_2(t)))_{t \geq 0}$$

2) Deduce that  $(X_{2,0}(t) - X_{2,0}(0))_{t \geq 0} \stackrel{(L_2)}{\equiv} (Y_{2,0}(t))_{t \geq 0}$



$$\stackrel{(L_2)}{\equiv} -\log(R(U, \beta_0))$$

