The development of modern mathematics in Mongolia

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1. Introduction

“A small nation with a big history” seems to be the briefest phrase that outlines the current state of Mongolia. This short essay focuses only on a tiny and recent part of the big history, namely the development of modern mathematics, which is mainly a 20th century phenomenon. It is my hope that the reader will not only have a chance to get acquainted with the existence of this development but also will be able to identify some features which make it unique. Nevertheless, the reader is warned that this is a brief account, which exclusively concentrates on the recent past. It aims to be informative but omits many details and excludes developments in other closely related fields such as mathematics education and applied mathematics. A systematic analysis from the perspective of scientific history is largely missing.

2. The beginnings

The first Mongolian university, the National University of Mongolia (NUM), was established in 1942. Initially, most of the university lecturers came from the USSR and the role of NUM concentrated on educating teachers. Accordingly, the faculty of pedagogy was one the first three faculties of NUM. One of its divisions was the Department of Mathematics, which offered mathematics courses, mainly directed to students in the Mathematics and Physics teachers program.

During 1942-1957 a four-year undergraduate program in Mathematics and Physics teaching was offered. Between 1957-1962 the program was separated into a Mathematics teaching program and Physics teaching program. In 1962 an undergraduate program in mathematics with five years of study was launched and it continued until 1994 when it was switched to a four year program. To date, many of the elder generation mathematicians are proud of the fact that they studied five years, which included many Saturdays as working days.

МУИС-ийн анхны багш нар (1948)

Figure 1: The first generation of Mongolian university lecturers, 1948

Shagdar Luvsantseren was amongst the first generation of students who entered a year after NUM’s establishment. He became the first Mongolian to obtain a Ph.D in mathematics, graduating in 1954 from Moscow State University (MSU). He was a probability theorist and at the MSU he studied under academic supervision of E. B. Dynkin. And thus, the Mongolian research agenda in mathematics started

with probability theory. Indeed, it went on to dominate the scientific agenda of Mongolian mathematics until the mid-nineties. Unfortunately, Prof. Shagdar Luvsantseren lost his eyesight around 1962, which made him less effective both in teaching and research.

The 1960s saw acceleration in the teaching and research of mathematics. In addition to regular seminars on probability and statistics, a seminar in algebra was started which eventually attracted many young talented students. During this period, at the university level, more specialized courses began to be offered to senior students and the first mathematics textbooks authored by Mongolian scholars were published. Moreover, at the high school level, first regional and then national mathematical Olympiads were organized. These later became an integrated part of mathematics education and research.

Л.Шагдар багшийн 50 насны ойн үеэр (1975)

Figure 2: 50'th birthday of Prof. Shagdar Luvsantseren

Officially, the Mongolian Mathematical Olympiad (MMO) started in 1965 and the first participation of Mongolia to the International Mathematical Olympiad (IMO) dates to 1964. Since then, in 44 participations, the Mongolian team got 2 gold medals, 19 silver medals, 56 bronze medals and 47 honorable mentions from the IMO. In 1969 the first time a problem proposed by a Mongolian mathematician, Abish Mekei (1940 - 2013), was elected as one of the IMO's six problems. Interestingly, during the IMO the original solution of this problem was improved by one of the contestants, Vladimir Drinfeld, the later Fields medalist. The first Mongolian gold medal from the IMO dates to 1997, which is 11 years ahead of Mongolia's first gold medals from the Olympic Games (in Judo and Boxing).

One of the first generation algebraists, Sanjmyatav Urjintseren (1926 - 2003), was a key figure in the Olympiad movement, and he acted as the chairman of the national Olympiad committee for 17 years. In 1994 he received the Paul Erdős Award for playing a significant role in the development of mathematical challenges at the national or international level and being a stimulus for the enrichment of mathematics learning. Aside from these successes, there is a caveat in this part of our mathematical history, namely that in 1980 Mongolia planned to host the IMO but unfortunately failed to do so and it is the only year in which the IMO was cancelled.

Figure 3: Sanjmyatav Urjintseren recieving the Paul Erdos Award, 1994.

3. From 1970 to 2000

Following Prof. Shagdar Luvsantseren, the second Mongolian Ph.D. was obtained in 1970 by Prof. Shagdar Damba, again in the field of probability theory, but this time from the University of Kyiv under the supervision of A. V. Skorokhod. The first Mongolian who specialized in algebra was Prof. Abish Mekei, who obtained a Ph.D from MSU in 1972, working on ring theory. The first specialist in analysis was Prof. Lkhamsuren Tsend who took his Ph.D from MSU in 1973, whilst the first in geometry was Prof. Radnaa Choijil who obtained his Ph.D from Kazan University in 1980. So, after probability theory, we can say that the next wave of mathematics, that came in the 1970s, offered three A's: namely Algebra, Analysis and Analytic geometry.

Around 1980s a close regional cooperation was established with the Novosibirsk State University and the Sobolev Institute of Mathematics. Dr. Gonchigdorj Radnaasumberel and Prof. Dashdorj Tserendorj were among the people who spent a considerable amount of time in Novosibirsk. As an example of the impression that was left, in a recent international conference, when a Mongolian mathematician approached Efim Zelmanov, he recognized the name "dorj" on his nametag and asked if he was a Mongolian (the full name of this person was Chuluundorj Bekh-Ochir).


Математикийн багш нар (1983)

Figure 4: The Math. department staff, 1983

That said, none of the "dorj"s defended their Ph.D thesis in Novosibirsk. The story of Prof. Dashdorj Tserendorj is somewhat complicated. He disagreed with his advisor on which problem he should work. After several years struggling without much support from his advisor, he finally managed to solve a problem of his own choice, which was initially proposed by K. A. Zhevlakov, and obtained Ph.D degree from the Moldavian Academy of Sciences in 1987; see problem No.57 in [4]. For many years, Prof. Dashdorj Tserendorj acted as the chairman of the MMO committee and as the Mongolian team leader in the IMO. In 1988 he published in Russian a result on the so called weak Schur numbers in a local journal called The Scientific Documents of the NUM; see [3]. In 2002 the same result was rediscovered and published by French mathematician Pierre Bornsztein, see [2]. In the literature, this result is usually attributed to the latter paper and not the former; see [8].

In 1969 the champion of the MMO in the 10^{th} grade (which was the highest grade) was a student named Haltar Damba, who came from the remote Hovd province in the far west of Mongolia. This is a rare event in the history of the Olympiad. It is the first time that someone outside of Ulaanbaatar, the capital city, won the championship. Haltar Damba went on to study mathematics at MSU and established a close relationship with well-known mathematicians such as Yu. M. Svirezhev and V. M. Tikhomirov. As a working mathematician, he developed several mathematical models directly related to the Mongolian nomadic lifestyle. These included, for example, horseracing and optimally locating nomads in a pastoral

territory. Legend says that, in the first year of his study at MSU, his teachers exemplified him as one of the best students. However, in the later years this situation was reversed as he started to drink too much. In 2009 he published his autobiography and memoirs in a local mathematical journal, called "Olonlog", in which he analyzed his own career as a mathematician and gave a fair account of his successes as well as his failures. In a sense, he wrote a Mongolian version of Hardy's apology; see [6].

Figure 5: During an international conference in Mongolia, 2006

In order to get someone specialized in the field of topology, Mongolia had to wait until 2002. Considering that topology was the forerunner of 20th century mathematics, this is a considerably long delay. This is partly due to the 1990's economic downturn when Mongolia abruptly abandoned communism and switched to a capitalist economy. These were very difficult times during which supporting daily life was a genuine struggle, and doing mathematics was certainly not the most profitable business.

4. Recent developments and some lessons from the past

From 2000 onwards the number of Mongolian Ph.D.s in mathematics from abroad increased sharply. Accordingly, the mathematics community started to become more diverse and it is likely that this trend will continue in the future. Increases in the quantity of returning students with a doctoral qualification were so sharp that some senior Mongolian mathematicians started to become concerned about their quality. In general, they say, mixing political and other motivations with academic research increases the possibility of type one and type two errors; namely, granting an academic degree to someone whose work is not qualified and not granting it to someone whose work is qualified. For sure, neither of the two is desirable.

To our knowledge, the first Mongolian who got a Ph.D from a High-Income country is Dr. Battsengel Baasanjav (Ph.D from the University of Tokyo in 1999). Since then a close collaboration with the University of Tokyo was established. Other institutions, with which continuing relations are held, include the ICTP (Trieste, Italy), the IMU-CDC and CIMPA (Nice, France).

Figure 6: The first generation of Japanese trained Ph.Ds

In any historical study, giving an exact account of past events is not enough as it then opens many questions such as "why did it happen in that way?" A moment of thought on the above account suggests investigation of the following three

issues, which are also relevant for the future.

- Lesson-1: There seems to be slow progress in opening new research branches, why? The first jump appeared within 12 years of establishing Mongolian academia (1942-1954). But the second jump occurs after 18-26 years (1954-1972, 1973, 1980) and the third jump after 22- 30 years (1972, 1980-2002). My hypothesis is that a version of Robert K. Merton's Matthew effect which is often summarized as "the rich get richer, the poor get poorer" emerged in the Mongolian mathematics community; see [10].
- Lesson-2: We tend to fail in advertising properly our ideas as it happened with the case of Prof. Dashdorj Tserendorj's result on weak Schur numbers. This is partly because Mongolia had a rather restricted international network. Indeed, in 1986, Prof. Dashdorj Tserendorj sent an abstract of his paper to the ICM, and was invited to come to California but he could not make it to the USA.
- Lesson-3: The social, political and economic upheaval around 1990 shows that the popular phrase "research in mathematics only needs pen and paper and nothing more" is not entirely true. Pen and paper are indeed necessary but certainly not sufficient. Indeed, without proper institutional and financial support, mathematics research can easily be abandoned.

5. Mongolia in the mathematics dictionary

Let me finish this essay by introducing two interesting cases where the reader might encounter Mongolia by name in the domain of mathematics.

- A graph obtained from the graph Cartesian product $P_m \times P_n$, where *n* is odd, by adding an extra vertex above the graph and joining every other vertex of the top row to the additional vertex is known as the **Mongolian tent graph**; see [5], [9]. To the best of our knowledge, this graph was introduced to the literature by Prof. Sin-Min Lee and it is clear that the resulting configuration resembles a Mongolian "ger."

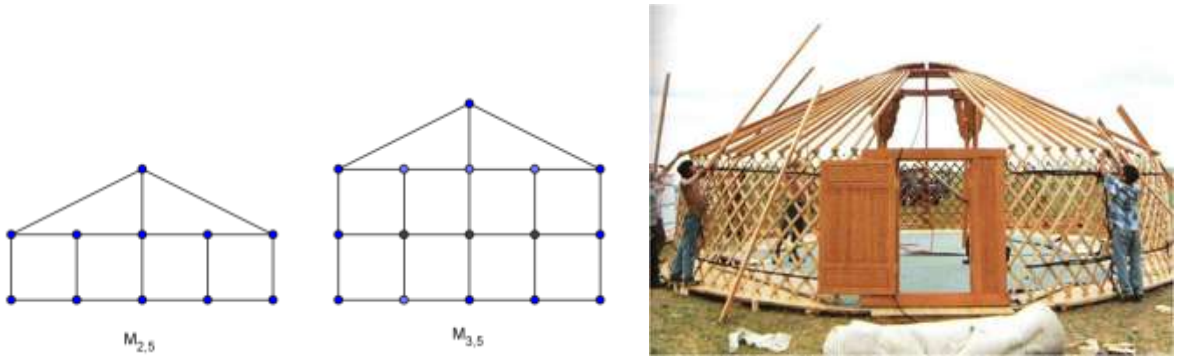

Figure 7: Mongolian tent graph vs. a real Mongolian ger

- In 31th MMO Dr. Adiyasuren Vandanjav proposed the following problem for a teacher's competition:

  Let $a_1, a_2, \ldots, a_n$ be positive numbers such that $a_1 \geq a_2 \geq \ldots \geq a_n$. Show that

$$\frac{a_1+a_2}{2} \cdot \frac{a_2+a_3}{2} \cdot \ldots \cdot \frac{a_n+a_1}{2} \leq \frac{a_1+a_2+a_3}{3} \cdot \frac{a_2+a_3+a_4}{3} \cdot \ldots \cdot \frac{a_n+a_1+a_2}{3}$$

  Despite its simplicity, this problem turned out to be rather challenging and no one could provide a complete solution when it was first posed. The inequality was later proved by some Russian mathematical Olympiad enthusiasts and it has since been referred to as a **Mongolian inequality**; see [1], [7].

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References: The main source of the historical account above is [11].

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