# THE PARADOXICAL NATURE OF THE PROPORTIONAL HAZARDS MODEL OF RANDOM CENSORSHIP*

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**Summary.** We show that the proportional hazards model of random censorship is too good to be true as measured by mean squared errors: for estimating the underlying distribution function $F(x)$ it is better to have a censored sample for a suitable expected censoring proportion than an uncensored full sample of the same size for any $x$ below the 0.56-quantile of $F$.

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## 1. INTRODUCTION

Let $X_1, X_2, \ldots, X_n$ be independent random variables with a *continuous* distribution function $F(x) = P\{X \leq x\}$, $x \in I\!R$. An independent sequence of independent random variables $Y_1, Y_2, \ldots, Y_n$ with distribution function $G$ censors them on the right, so that at each stage $n$ we can only observe $Z_j = \min(X_j, Y_j)$ and $\delta_j = I\{X_j \leq Y_j\}$, $j = 1, \ldots, n$, where $I\{A\}$ stands for the indicator of an event $A$. Let $H$ be the distribution function of $Z$, where $(Z, \delta) = (Z_1, \delta_1)$, so that $1 - H = (1 - F)(1 - G)$ and $p = P\{\delta = 1\} = \widetilde{H}(\infty)$ is the expected proportion of uncensored observations, where $\widetilde{H}(x) = P\{Z \leq x, \delta = 1\} = \int_{-\infty}^{x} [1 - G_-(y)]\, dF(y)$, $x \in I\!R$, with the left-continuous version $G_-$ of $G$. This is the widely used random censorship model, in which considerations are centered around properties of the celebrated Kaplan – Meier product-limit estimator $\hat{F}_n$ of $F$. The literature is enormous; for many of the standard references and the latest developments the reader is referred to Csörgő (1997).

The special proportional hazards submodel of random censorship is defined by the existence of a positive constant $c$ such that $1 - G(x) = [1 - F(x)]^c$ for all $x \in I\!R$, so that the censoring and censored cumulative hazard functions $\Lambda_G = -\log(1 - G)$ and $\Lambda_F = -\log(1 - F)$ are proportional: $\Lambda_G = c\Lambda_F$. This submodel, in which $p = 1/[1 + c]$ and $1 - F = [1 - H]^p$, has been popular for theoretical purposes since it allows calculations for the investigation of properties of $\hat{F}_n$ which are easier to derive and are more

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readily interpretable than in the general censorship model. It has also been repeatedly recommended as an intuitively appealing and potentially useful applied scenario. This orientation in the literature has become particularly pronounced with the appearance of the Abdushukurov – Cheng – Lin estimator $F_n(x) = 1 - [1 - H_n(x)]^{p_n}$ of $F(x)$, where $p_n = n^{-1}\sum_{j=1}^{n}\delta_j$ and $H_n(x) = n^{-1}\sum_{j=1}^{n} I\{Z_j \le x\}$ is the sample distribution function of the minimum observations, $x \in I\!R$. Most of the relevant literature up to 1987 has been summarized by Csörgő (1988), though it turned out that an unpublished preprint of Hollander, Proschan and Sconing (1985) also derived $F_n$, and investigated some of its appealing properties, independently of the papers by Abdushukurov (1987) and Cheng and Lin (1987). (Due to a mistake of one of the present writers, the submodel is often inappropriately referred to as the Koziol – Green model; cf. the corresponding remarks in Csörgő (1988).) The literature on various aspects of estimation and testing in the proportional hazards submodel and its slight extensions or variants has become quite sizable following 1988; see, for example, Csörgő and Mielniczuk (1988), Ghorai (1989 a, b, 1991 a, b), Gijbels and Veraverbeke (1989), Hollander and Peña (1989), Dikta and Ghorai (1990), Mi (1990, 1996), Gijbels and Klonias (1991), Rao and Talwalker (1991), Ghorai and Pattanaik (1991, 1993), Beirlant, Carbonez and van der Muelen (1992), Dhar (1992), Herbst (1992 a, b, 1993, 1994), Janssen and Veraverbeke (1992), Stute (1992), Peña and Rohatgi (1993), Veraverbeke (1994) and Dikta (1995).

Quite naturally, most of these papers are concerned with the superiority of procedures for estimation and testing within the submodel which are based on $F_n$ rather than the product-limit estimator $\hat{F}_n$. Indeed, the success is spectacular at times, and it is in this sense that the submodel is usually meant to represent a form of informative censoring. In the simplest situation, if the submodel holds, the asymptotic squared error of $F_n(x)$ is strictly less than that of $\hat{F}_n(x)$ for any $x \in I\!R$ for which $0 < F(x) < 1$; cf. Abdushukurov (1987), Cheng and Lin (1987), Hollander, Proschan and Sconing (1985), or Csörgő (1989). Of course, if the data $(Z_1, \delta_1), \ldots, (Z_n, \delta_n)$ from the general censorship model do not follow the proportional hazards submodel, these procedures may be very bad: $F_n$ still estimates $1 - [1 - H]^p$, but this is no longer $F$.

How frequent is the submodel in practice? It holds if and only if $Z$ and $\delta$ are independent (cf. Csörgő (1988) for references), and this fact is what makes the superior $F_n$ possible in it. However, the same fact already makes its regular occurrence in real data doubtful. Csörgő (1989) has derived some omnibus large-sample tests for testing for the submodel, which have quite reasonable small-sample properties. Going through a number of published data sets available at the time, he did find one famous data set which follows the proportional hazards submodel quite closely: the Channing House data discussed by Efron (1982). His findings are fully corroborated by Henze (1993). This is sometimes taken as an encouraging sign that the submodel may indeed be practically useful quite widely. But, to the best of our knowledge, this is the *only* censored data set known to be of the proportional hazards type.

The presence of random censorship is naturally associated with loss of information: one cannot observe the full sample $X_1, \ldots, X_n$ uncensored, so one loses information to properly estimate $F(x)$ by the unavailable empirical distribution function $F_n^*(x) = n^{-1} \sum_{j=1}^n I\{X_j \le x\}$; instead, one is relegated to the use of $\hat{F}_n(x)$ or $F_n(x)$, $x \in I\!R$. This is well understood for the relation between $\hat{F}_n$ and $F_n^*$ for the general model. Here $\sqrt{n}\,[\hat{F}_n(x) - F(x)]$ is asymptotically $\mathcal{N}(0, v^2(x))$ for some function $v(\cdot)$, while, if the full sample is known, $\sqrt{n}\,[F_n^*(x) - F(x)]$ is asymptotically $\mathcal{N}(0, F(x)[1 - F(x)])$ as $n \to \infty$, where $\mathcal{N}(0, \sigma^2)$ denotes the normal distribution with mean 0 and variance $\sigma^2$, and $v^2(x)/\{F(x)[1 - F(x)]\} > 1$ for all meaningful $x$ if $p < 1$; cf. Csörgő (1997) for instance.

The aim of the present note is to point out that, with goodness measured by mean squared errors, this is not so with the proportional hazards submodel of random censorship: up to a strange quantile beyond the median, one may be better off having a censored sample than the uncensored full sample! This paradoxical nature of the model partly explains its rarity: it is too informative, it is in fact too good to be frequently true in practice.

## 2. THE PARADOXICAL NATURE OF THE MODEL

Suppose the great Maker* shows Peter the censored data $(Z_1, \delta_1), \ldots, (Z_n, \delta_n)$ with the extra information that it is from a proportional hazards model of random censorship with some untold $p \in (0, 1)$. "*Himself all-wise, all-powerful* and *good*," he however reveals to Paul, his favorite to all appearance, the full data set $X_1, \ldots, X_n$ uncensored.

Suppose first that Peter and Paul wish to estimate the density function $f(\cdot)$ of $F(\cdot)$, assumed to exist for the moment. Peter then forms the obvious kernel estimator $f_n(x)$ based on $F_n$, while Paul, using the same kernel and bandwidth $b_n$, forms the classical kernel estimator $f_n^*(x)$ based on the full sample distribution function $F_n^*$ known to him. Let $F^{-1}(s) = \inf\{y : F(y) \ge s\}$, $0 < s < 1$ denote the corresponding quantile function. Under standard smoothness conditions and the usual conditions on $b_n$ to make both estimators asymptotically normal with zero mean, Csörgő and Mielniczuk (1988) have

* Abraham de Moivre: *The Doctrine of Chances*, Third Edition, London, 1756; page 252.

shown that the limiting variance of $f_n(x)$ is strictly smaller than the limiting variance of $f_n^*(x)$ for any $x < F^{-1}(1-e^{-1})$, provided Peter is allowed to use a suitable $p$ depending on $x$. Interestingly, the optimal $p = p_\diamond(x)$ for which the ratio of the former to the latter is the smallest is $\Lambda_F(x)$, and this minimal ratio is given in Figure 1 there.

This finding may not be impressive in that that it is achieved by suitable choices of the bandwidth parameters that make it possible to ignore the bias of the estimators asymptotically; the bias is present in all finite samples for both estimators.

Suppose now that Peter and Paul set out to estimate $F$. Naturally, Peter will use $F_n$ and Paul will use $F_n^*$, both asymptotically optimal, with generally optimal small-sample properties, from their respective vantage points. Then $\sqrt{n}\,[F_n(x) - F(x)]$ is asymptotically $\mathcal{N}(0, \sigma_F^2(x))$ and $\sqrt{n}\,[F_n^*(x) - F(x)]$ is asymptotically $\mathcal{N}(0, F(x)[1-F(x)])$ as $n \to \infty$, where, setting $\log^2 y = [\log y]^2$, $y > 0$,

$$\begin{aligned}\sigma_F^2(x) &= [1-F(x)]^2 \left\{ p^2 \frac{H(x)}{1-H(x)} + p(1-p)\log^2(1-H(x)) \right\} \\ &= [1-F(x)]^2 \left\{ p^2 \frac{1-[1-F(x)]^{1/p}}{[1-F(x)]^{1/p}} + \frac{1-p}{p}\log^2(1-F(x)) \right\}\end{aligned}$$

for any $-\infty \le \lim_{s\downarrow 0} F^{-1}(s) = F^{-1}(0) < x < F^{-1}(1) = \lim_{s\uparrow 1} F^{-1}(s) \le \infty$, from Abdushukurov (1987), Cheng and Lin (1987), Hollander, Proschan and Sconing (1985), or from (2.6) in Csörgő (1988).

Therefore, the ratio $R(F(x), p) = \sigma_F^2(x)/\{F(x)[1-F(x)]\}$ is a measure of the relative asymptotic performance of Peter and Paul, where

$$\begin{aligned}R(s,p) &= p^2\,\frac{1-s}{s}\left[\frac{1}{(1-s)^{\frac{1}{p}}} - 1\right] + \frac{1-p}{p}\,\frac{1-s}{s}\,\log^2(1-s) \\ &= p^2\left[\frac{1}{s(1-s)^{\frac{1}{p}-1}} - \frac{1-s}{s}\right] + \frac{1-p}{p}\,\frac{1-s}{s}\,\log^2(1-s), \quad 0 < s < 1.\end{aligned}$$

This would be expected to be greater than $1$, on the basis of superficial general intuition that censoring is loss of information, with $R(1,p) = \lim_{s\uparrow 1} R(s,p) = \infty$ for any $p \in (0,1)$. Indeed, the latter is true from the second formula: Paul is asymptotically far better off than Peter at large quantiles $x = F^{-1}(s)$. However, from the first formula, $R(0,p) = \lim_{s\downarrow 0} R(s,p) = p$ for every expected proportion $p \in (0,1)$ of uncensored observations. Hence for the $s^*(p)$-quantile $F^{-1}(s^*(p))$ of the estimated distribution, where $s^*(p) \in (0,1)$ is the unique quantity $s$ for which $R(s,p) = 1$, Peter is asymptotically better off than Paul for all $x$ in the whole half-line $(-\infty, F^{-1}(s^*(p)))$. In fact, $s^*(0.1) \approx 0.09324$, $s^*(0.3) \approx 0.24555$, $s^*(0.5) \approx 0.36340$, $s^*(0.7) \approx 0.45619$, $s^*(0.9) \approx 0.53038$, $s^*(0.95) \approx$

0.55658, $s^*(0.99) \approx 0.55896$, $s^*(0.999) \approx 0.56167$, and, with the $s^*(p)$ values projected down to the horizontal axis, Figure 1 contains the curves $R(s,p)$, $0 \le s \le s^*_{1.2}(p)$, for $p = 0.1$, $p = 0.3$, $p = 0.5$, $p = 0.7$, $p = 0.9$ and $p = 0.95$, where $s^*_{1.2}(p) > s^*(p)$ is the unique quantity $s$ for which $R(s,p) = 1.2$. The horizontal line at height 1, from which the projections are done, is of course the graph of $R(s,1) = 1$, $0 \le s \le 1$, for $p = 1$. Our new intuition now, given the evidence, is that when estimating $F(x)$ at a small quantile $x$, proportional hazards censoring is advantageous since a small fraction of large observations gets replaced by almost the same but smaller observations and their knowledge is independent of the knowledge of whether they are censored or not.

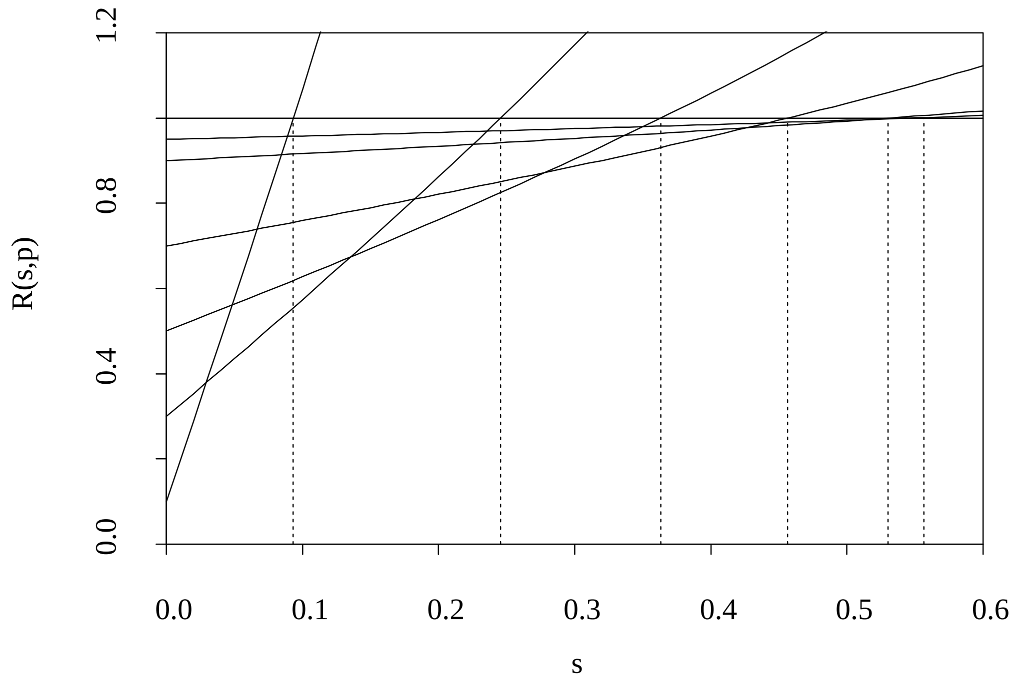


**Figure 1.** The functions $R(s,p)$, $0 \le s \le s^*_{1.2}(p)$, where $R(0,p) = p$, for $p = 0.1$, $p = 0.3$, $p = 0.5$, $p = 0.7$, $p = 0.9$, $p = 0.95$ and $p = 1$.

Peter's gain in asymptotic efficiency is considerable for small $p$, albeit only for small quantiles. While his interval of advantage is longer, his gain is not that much for a large $p$. Suppose, however, that for each $s$-quantile $x = F^{-1}(s)$, $s \in (0,1)$, the *all-wise* Maker allows Peter to choose the best possible $p = p_*(s) \in (0,1]$, to "compensate" him for his initial illusory disadvantage. (Peter still does not know the numerical value of $p_*(s)$, he only obtains the proportional-hazards censored data as before with this $p = p_*(s)$ to estimate $F(x)$ for $x = F^{-1}(s)$. Otherwise he would use the *even better* estimator $\widetilde{F}_n(x) = 1 - [1 - H_n(x)]^{p_*(s)}$ for this $x$.) Thus, for each $s \in (0,1)$, the unique quantity

$p_*(s) \in (0,1]$ is for which $R_*(s) = R(s, p_*(s)) < R(s,p)$ for any $p \in (0,1]$, $p \neq p_*(s)$, where the choice $p_*(s) = 1$ for some $s \in (0,1)$ forces $\delta_1 = 1, \delta_2 = 1, \ldots$ almost surely; the choice of the same uncensored sample that Paul has. Peter now is wise enough himself not to choose hastily $p = 1$ for all $s \in (0,1)$. By differentiation, for any fixed $s \in (0,1)$ his choice $p_*(s) \in (0,1]$ is the $p \in (0,1]$ for which $dR(s,p)/dp = 0$, that is, for which

$$2p - 2p(1-s)^{1/p} + \log(1-s) - p^{-2}(1-s)^{1/p} \log^2(1-s) = 0.$$

The critical quantile $s_* = \sup\{s \in (0,1) : R_*(s) < 1\}$ below which Peter has an advantage by using data from a proportional hazards censoring model is obtained by letting $p \uparrow 1$ in the equation above: it is the solution $s \in (0,1)$ of the equation

$$2s + \log(1-s) - (1-s)\log^2(1-s) = 0.$$

Alternatively, $s_* = \lim_{p\uparrow 1} s^*(p)$. We obtain $s_* \approx 0.56197$. For $x = F^{-1}(s)$ with $s \geq s_*$, Peter wants to use $p_*(s) = 1$, i.e. the uncensored full sample that Paul has. Figure 2 depicts Peter's optimal choice $p_*(s)$, in the solid line, and his gain with this choice is shown by the corresponding graph of the optimal ratio $R_*(s)$, in the dashed line, $s \in (0,1)$. The gain for smaller quantiles is considerable.

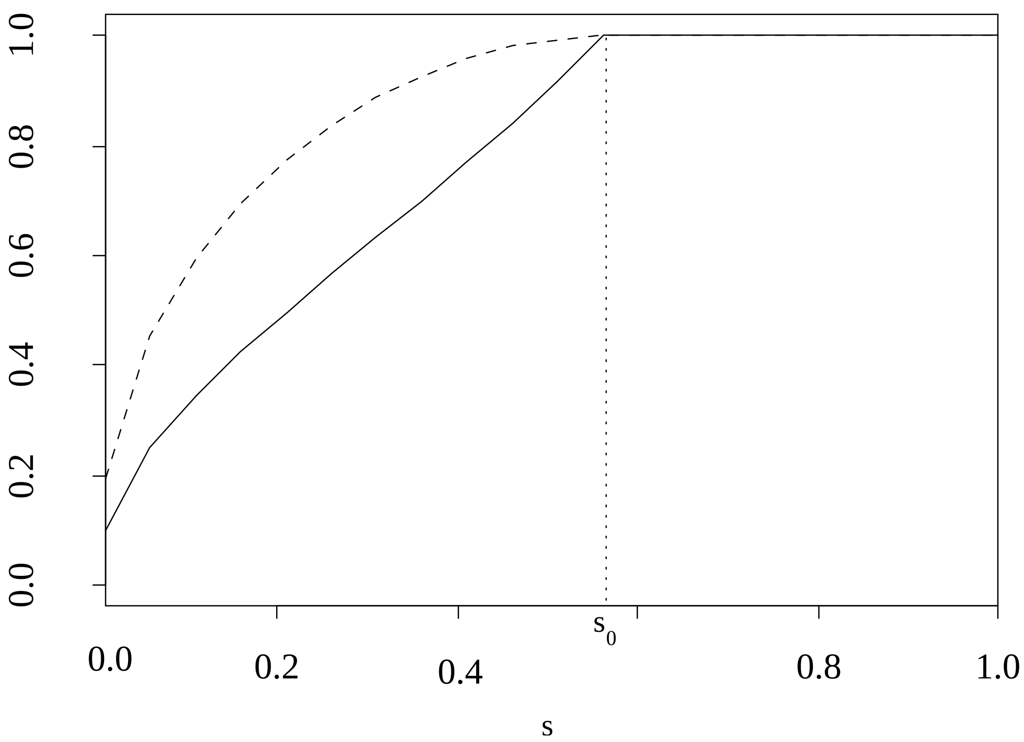


**Figure 2.** The functions $p_*(s)$ (solid curve) and $R_*(s)$ (dashed curve), $0 < s < 1$.

It may be thought at this point that Peter can use his advantage, given to him by the MAKER and represented in Figure 2, only for very large sample sizes. The above considerations are asymptotic in nature and disregard issues of bias: while for each fixed $n$ the mean squared error of Paul's unbiased estimator $F_n^*(x)$ is $M_n^*(x) = F(x)[1-F(x)]/n$, the corresponding mean squared error $M_n(x) = [E(F_n(x) - F(x))]^2 + n^{-1}\mathrm{Var}(F_n(x)) = [E([1 - F_n(x)] - [1 - F(x)])]^2 + n^{-1}\mathrm{Var}(1 - F_n(x))$ for Peter's biased estimator $F_n(x)$ is far more complicated. Since $H_n(x)$ and $p_n$ are independent, conditioning on the latter, we obtain $E([1 - F_n(x)]^\alpha) = B_{n,\alpha}(F(x))$ for every $\alpha > 0$, where

$$B_{n,\alpha}(s) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} \sum_{j=0}^{n} \left(\frac{j}{n}\right)^{k\alpha/n} \binom{n}{j} (1-s)^{j/p} \big[1 - (1-s)^{1/p}\big]^{n-j}$$

for all $s \in (0,1)$. This was noticed already by Hollander, Proschan and Sconing (1985). Hence the ratio $R_n(F(x), p) = M_n(x)/M_n^*(x)$ for sample size $n$, corresponding to the limiting $R(F(x), p)$ above can be obtained by substituting $s = F(x) \in (0,1)$ into

$$R_n(s,p) = \frac{n[B_{n,1}(s) - (1-s)]^2 + [B_{n,2}(s) - B_{n,1}^2(s)]}{s(1-s)}\,.$$

One can then define $s_*(n) = \sup\{s \in (0,1) : R_n(s, p_*(s;n)) < 1\}$, for sample size $n$, as the analogue of $s_*$ above, where $p_*(s;n)$ is the value which minimizes $R_n(s,p)$ for any given $s \in (0,1)$.

We find $s_*(2) \approx 0.18767$, $s_*(3) \approx 0.39991$, $s_*(4) \approx 0.47482$, $s_*(5) \approx 0.51141$, $s_*(6) \approx 0.53226$, $s_*(7) \approx 0.54500$, $s_*(8) \approx 0.55301$, $s_*(9) \approx 0.55812$, $s_*(10) \approx 0.56139$, $s_*(11) \approx 0.56348$, $s_*(12) \approx 0.56480$, $s_*(13) \approx 0.56563$, $s_*(14) \approx 0.56615$, $s_*(15) \approx 0.56645$, $s_*(16) \approx 0.56661$, $s_*(17) \approx 0.56668$, $s_*(18) \approx 0.56670$, $s_*(19) \approx 0.56667$, $s_*(20) \approx 0.56685$. Hence Peter may estimate the median better than Paul already with sample size $n = 5$. Furthermore, not only the qualitative findings for $n = \infty$ set in already at $n = 10$, but in fact $s_*(n)$ overshoots $s_* = s_*(\infty) \approx 0.56197$ a little for $n \geq 11$. Thus, when estimating $F(x)$ for any $x$ below the 0.56-quantile, Peter will be better off with a censored sample from a suitable proportional hazards model than Paul with the full uncensored sample for any sample size $n \geq 10$.

We do not have an intuitive explanation neither for the value of the critical quantile $s_\diamond = 1 - e^{-1} \approx 0.63212$ for density estimation, nor for the value of the critical quantile $s_* \approx 0.56197$ in the present situation when estimating a distribution function. Regardless of the values themselves, we do not even have any heuristics to explain why $s_* < s_\diamond$.

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