The Law of the Supremum of a Stable Lévy Process

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Let $X = (X_t)_{t \geq 0}$ be a stable Lévy process of index $\alpha \in (1, 2)$ with no negative jumps, and let $S_t = \sup_{0 \leq s \leq t} X_s$ denote its running supremum for $t > 0$. We show that the probability density function f_t of S_t can be characterised as the unique solution to a weakly singular Volterra integral equation of the first/second kind, or equivalently, as the unique solution to a first-order Riemann-Liouville fractional differential equation satisfying a boundary condition at zero. This yields an explicit series representation for f_t. Recalling the familiar relation between S_t and the first entry time τ_x of X into $[x, \infty)$, this further translates into an explicit series representation for the probability density function of τ_x.

The purpose of the talk is to present and discuss the proof, paying a particular attention to the three integral equations which characterise f_t, and examining their interplay.

This is a joint work with V. Bernyk and R. C. Dalang (both of Lausanne).