

The LQ-problem for abstract hyperbolic systems

Francesca Bucci
Università degli Studi di Firenze
Dipartimento di Matematica Applicata
Via S. Marta 3, I-50139 Firenze, ITALY
fbucci@dma.unifi.it

Abstract

Optimal control problems with quadratic functionals for (linear) Partial Differential Equations (PDEs) over a finite or infinite time horizon and related differential/algebraic Riccati equations have been the object of research during the past 20 years. A rather complete theory, based on an operator theoretic approach, has been developed when the cost functional is *coercive*. This theory distinguishes few basic classes of abstract dynamics, primarily parabolic-like dynamics and hyperbolic-like dynamics, which model most PDEs with boundary/point control of interest.

Recently the problem of optimization of more general quadratic cost functionals (the so called “non-standard” LQ-problem) has been investigated in the case of boundary control systems. In fact it is well known that *indefinite* (quadratic) functionals naturally arise in several fields of control theory, e.g. in the study of stability properties of *non-linear* feedback systems.

In this lecture we present both an overview and recent results about the non-standard LQ-problem for an abstract class of dynamics $y' = Ay + Bu$ with *unbounded* control operator B , which models a variety of hyperbolic (hyperbolic-like) PDEs with boundary/point control. Our analysis will include the case when the operator $e^{tA}B$ satisfies a suitable singular estimate, yet the s.c. semigroup e^{tA} is not analytic. Motivation to study this latter class of dynamics comes from coupled systems of PDEs, with hyperbolic/parabolic coupling, which mathematically model a variety of engineering problems involving interactive structures.

We shall discuss results pertinent to the optimal control problem over a finite time horizon and related (integral) Riccati equation. Stability properties of the system, in connection with the infinite time horizon problem, are under investigation.