

# Backstepping for partial differential equations: making infinite recursions convergent

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## Abstract

We consider feedback transformations of the backstepping/feedback linearization type that have been prevalent in finite dimensional nonlinear stabilization, and, with the objective of ultimately addressing nonlinear PDE's, generate the first such transformations for a linear PDE that can have an arbitrary finite number of open-loop unstable eigenvalues. These transformations have the form of recursive relationships and the fundamental difficulty is that the recursion has an infinite number of iterations. Naive versions of backstepping lead to unbounded coefficients in those transformations. We show how to design them such that they are sufficiently regular (not continuous but  $L^2$ ). We then establish closed-loop stability, regularity of control, and regularity of solutions of the PDE. We illustrate and extend the concept to a nonlinear model of a combustion instability in a solid propellant rocket, for which we design a stabilizing boundary controller.