Group Theory: Sheet 2

©G. C. Smith 2001

The copyright is waived for non-profit making educational purposes.

1. Let $G = \mathrm{Sym}(\{1,2,3,4\})$. Consider the subgroup $H = \{\mathrm{id}, (1,2,3), (1,3,2)\}$. Explicity describe the sets G/H and $H\backslash G$ (i.e. list the left and right cosets of H in G).

2. Suppose that $H \leq K \leq G$. Let T be a left transversal for H in K, and let S be a left transversal for K in G.

   (a) Suppose that $t, t' \in T$ and $s, s' \in S$. Show that $st = s't'$ if and only if both $s = s'$ and $t = t'$.

   (b) Show that ST is a left transversal for H in G.

   (c) Deduce that $|G : H| = |G : K| \cdot |K : H|$.

3. Suppose that G is a group with a subgroup H. Suppose that T is a left transversal for H in G. Does it follow that T must also be a right transversal for H in G? Justify your answer.

4. Consider two non-parallel lines in a Euclidean plane. Let one of the angles between these lines be θ. Show that the composition of the reflections in this line is a rotation about the point of intersection. The rotation is through what angle?

5. Let H, K be subgroups of G. Suppose that $x, y \in G$ and $Hx = yK$. Does it follow that $H = K$? Justify your answer.