

BOUNDARY INTEGRAL EQUATION METHODS FOR HIGH FREQUENCY SCATTERING PROBLEMS

Investigators

Professor Ivan Graham, and Professor Valery Smyshlyaev, University of Bath.

Professor Simon Chandler-Wilde and Dr Stephen Langdon, University of Reading

Objectives of Project

The overall aim of the project is to develop numerical algorithms (based on boundary integral equation methods, supported by mathematical analysis and implemented and tested on important applications) for the accurate simulation of high frequency scattering problems. The novelty, difficulty, and practical significance of the proposal is that we aim to achieve a computational cost which does not increase (or increases only mildly) as the frequency increases even though the solution and integral operators become increasingly oscillatory.

To achieve this overall aim our sub-objectives are:

A. To construct hybrid approximation spaces which combine traditional finite-element-type representations with information derived from high frequency asymptotics, and to prove by mathematical analysis and demonstrate by numerical experiments that these spaces represent the solution with a number of degrees of freedom which grows at worst mildly with the frequency. To achieve these aims we shall need to use non-classical "uniform asymptotics" theory, as traditional geometric optics approaches will not be sufficiently powerful.

B. To develop quadrature/asymptotic methods for the computation of the oscillatory multiple integrals which arise in the resulting stiffness matrix entries, after the hybrid approximation spaces are used as the basis for a Galerkin/collocation method.

C. To establish the stability of these novel Galerkin/collocation methods, understand their conditioning, and establish a priori error estimates, with emphasis on the difficult aspect of behaviour as the frequency gets very large, and understanding the dependence of this behaviour on the frequency.

D. To develop appropriate fast methods to solve the dense linear systems that the new Galerkin methods will produce.

E. To produce high quality public domain software, available via a dedicated web site, which showcases the new algorithms applied to model problems (including radar scattering benchmark problems), which demonstrates the superiority of the new methods, and which serves as a source of prototypes for future code development. To apply the software to problems relevant to our project partners, BAE Systems, the Institute of Cancer Research (ICR), the Met Office and Schlumberger, all of whom are interested in high frequency acoustic and/or electromagnetic scattering problems.

The objectives will be achieved by PDRA appointments at each of Reading and Bath, and the dedicated research time of each of the four investigators, supplemented by short visits of three key visiting researchers.

The PDRAs will be trained in modern methods in high-frequency asymptotics, applied analysis, numerical analysis and scientific computing, through a programme of visits between the lively groups in Bath and Reading, thus giving them access to all four investigators and three visiting researchers (M. Ganesh, Colorado, V. Dominguez, Navarra and R. Hiptmair, ETH Zuerich (Chair of the Steering Committee)) and providing them with a unique combination of skills. The PDRA training programme will also include events like the Winter School in Hierarchical matrices at Leipzig, graduate courses at the two institutions and research visits/workshops in V.M. Babich's world-leading diffraction group in St Petersburg.