

BOUNDARY INTEGRAL EQUATION METHODS FOR HIGH FREQUENCY SCATTERING PROBLEMS

Investigators

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Summary of Project (for a non-specialist reader)

This project concerns the invention, analysis and implementation of new numerical methods for computationally simulating high frequency wave scattering problems. These problems have diverse applications, for example in modelling radar, sonar, acoustic noise barriers, medical ultrasound, and scattering of radiation by atmospheric particles. Our research is supported by four industrial/research organisations who comprise a steering committee and will provide motivating physical applications for the project.

The chief technological difficulty which we face in the project is that of computing accurately wave solutions which are highly oscillatory (i.e. varying very quickly). Standard approximation techniques usually break the domain of the problem up into small "elements", and use simple (e.g. polynomial) approximations on each element. Then it is known that about 5-10 elements are required in each wavelength to achieve acceptable accuracy, and so the computational work required grows at least in proportion to the frequency of the wave (and often faster than this). In this sense the methods are termed "not robust" as frequency increases.

We are going to devise, analyse and implement new robust methods for which the cost to obtain a fixed accuracy is bounded (or at worst grows very slowly) as the frequency increases. The programme involves solving problems not only of approximation of highly oscillatory solutions, but also (and this is often harder) analysing the stability and conditioning (i.e. "sensitivity") of the equations which govern them.

The chief device which we will use to achieve our objective is the great body of asymptotic techniques for high frequency wave phenomena, some of which which are well-known in the mathematics and physics communities but which have so far been very little used in numerical computation. Part of our project will involve the derivation of new asymptotic analyses and putting them in a form suitable for use in numerical analysis. Scattering problems will be reformulated in such a way that high frequency parts of the solution are handled explicitly (and thus exactly), leaving only the approximation of low frequency components which can be done with low cost. This approach leads to new, challenging and deep problems in consistency, stability, conditioning and numerical integration which must be solved before the robustness of the methods can be rigorously determined. Some of the problems which we face require applying technology which we know will work because of our earlier studies; others require a significant element of risk and a spirit of adventure.