

HGS Heidelberg Compact Course

“Computational Methods for Uncertainty Quantification”

Exercise Sheet 2

7. (Stochastic Collocation Methods – Slide 18):

- (a) The Matlab function `g_data(n)` (which is provided on my website) computes the Gauss-Legendre quadrature points and weights for the interval $(-1, 1)$. Transform and tensorise this set of points and hence write a program that evaluates \mathcal{G}_M at the Gauss points for user-defined values of M and n and then evaluates the integral (resp. expected value). Study the convergence with respect to n . What do you observe?
- (b) You are also given a set of model codes for the lognormal diffusion problem in 1D (again available on my website). Study the codes and experiment with the different methods. In particular, study the stochastic collocation code (based on Gauss-Legendre points mapped to the entire real line via the inverse CDF for the normal distribution). How fast does it converge for this problem? How does the cost grow with dimension s ? Compare to the different Monte Carlo codes (MC, QMC, MLMC, MLQMC).

8. (Quasi-Monte Carlo Methods – Slide 36):

- (a) Use the file `lattice-38005-1024-1048576.5000.txt` (provided on my webpage; you can also download it from Kuo’s webpage web.maths.unsw.edu.au/~fkuo/lattice/index.html). It contains a generating vector for a rank-1 lattice rule with equal weights $\gamma_j = 0.05$ to construct a set of QMC points on the unit square $[0, 1]^2$. Randomise and use this set to approximate $\mathbf{E}[u_1(T)]$ in the predator-prey example and compare the convergence of this QMC rule with the convergence of your other codes.
- (b) As part of the model codes for the lognormal diffusion problem in 1D you will also find a QMC code there. Experiment also with that code.