MA20010: Exam Feedback 2006/07

Question 1. Students did very well on this question. The average was almost 70%. You all managed very well to manipulate with curvilinear coordinates. However, be careful when you are asked to prove that something is true *"iff"* (i.e. *if and only if*). You need to show both implications, i.e. $a = b \;\Rightarrow\;$ the OCCs are orthogonal and the OCCs are orthogonal $\Rightarrow\; a = b$. There were also some minor manipulation errors and some wrong applications of the addition theorems.

Question 2. Most students were able to recall the Divergence Theorem with all assumptions correctly (Part (a)). Part (c), which included a hint about how to prove the identity of the two integrals, also was okay for most of you. Be careful, however, when manipulating with the Del-operator ∇; you cannot simply apply vector identities like $\mathbf{a}\cdot(\mathbf{b}\wedge\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\wedge\mathbf{a})$, because ∇ does not commute with a vector field, i.e. $\nabla\wedge\mathbf{F} \neq \mathbf{F}\wedge\nabla$ and so $\nabla\cdot(\mathbf{a}\wedge\mathbf{b}) \neq \mathbf{a}\cdot(\mathbf{b}\wedge\nabla)$. (The RHS doesn't even make any sense.) Also, and more severely, some people still think it is possible to <u>divide</u> by a vector, e.g. $\mathbf{a}\cdot\mathbf{F} = \mathbf{a}\cdot\mathbf{G} \;\Rightarrow\; \mathbf{F} = (\mathbf{a}\cdot\mathbf{G})/\mathbf{a} = \mathbf{G}$ where $\mathbf{a}$, $\mathbf{F}$ and $\mathbf{G}$ are vectors or vector fields. This is not possible and a <u>very serious</u> mistake for a second year Mathematics student!! Unfortunately, overall the performance on this question was quite poor though (i.e. 50% on average), since many of you couldn't recall the proof of Part (b) without a hint. (I did think that giving the hint in Part (c) would be enough to prompt you on the proof for Part (b) also.) The performance in Part (d) depended largely on whether you were able to recall the Helmholtz Theorem. For those who did remember it , this part of the question was very easy.

Question 3. Again the performance on this question was rather poor, but it varied considerably, with some students doing very well (Average: $\sim$50%). Part (a)(i) was okay. Almost everybody managed to recall what irrotational means. However, some people still can't distinguish between scalar and vector fields. In Part (a)(ii) I wanted a detailed proof of the Fundamental Theorem of Calculus for vector fields (as done on one of the problem sheets). Some people skipped too many steps. In particular, I really wanted to see whether you could apply the chain rule and the FTC for scalar functions correctly. The deduction that $\nabla\phi$ is conservative also caused problems. Please note again that for a closed curve $\mathbf{r}(\mathbf{t_0}) = \mathbf{r}(\mathbf{t_e})$ and <u>not</u> $t_0 = t_e$! In Part (b)(i) it was not possible to avoid parametrising the curves $\mathcal{C}_1$ and $\mathcal{C}_2$. Some students struggled with this, although they are simple straight lines between two points that are given! A further point to make is, that some of you still forget what a dot-product of two vectors is. In Part (b)(ii) you did have to actually give the required curves; and in deducing that $\mathbf{F} = \nabla\psi$, I came across things like $\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$ instead of $\mathbf{F} = \mathbf{F_1}\mathbf{i} + \mathbf{F_2}\mathbf{j} + \mathbf{F_3}\mathbf{k}$. Finally in Part (c) you have to be more careful in using the results from (a) and (b). Those results are all <u>implications</u> and <u>not equivalences</u>. The most common and crucial error in this part, however, was that apparently

$$\int\!\!\int_S (\nabla\wedge\mathbf{F})\cdot\mathbf{dS} = \mathbf{0} \quad\Leftrightarrow\quad \nabla\wedge\mathbf{F} = \mathbf{0}\ .$$

This is <u>not</u> correct. Only "$\Leftarrow$" is true!! Note that $\int\!\int_S (\nabla\wedge\mathbf{F})\cdot\mathbf{dS} = \mathbf{0}$ only implies that $\nabla\wedge\mathbf{F}$ is orthogonal to $\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is the unit outward normal to S.

Question 4. Unfortunately, most students didn't revise this part of the lectures (as usual). Less than a quarter of you attempted this question. The ones that had revised and did attempt this question not just because they were forced to, however, performed rather well (average $\sim$60%). In solving the eigenproblem (for $X(x)$) some possible solutions were lost on the way, i.e. many of you forgot the case $n = 0$ and didn't obtain $\sin(n\pi x)$ as a possible solution. Some crucial details in the method were also lacking in some cases. The Y-problem was generally okay, but again, the case $n = 0$ was often forgotten. (It differs from the other cases, i.e. $Y_0(y) = A_0\, y$ and not $Y_0(y) = A_0 \sinh(\mathrm{n}\pi\mathrm{y})$!) In a few cases the X- and the Y-problems were interchanged. Most/all of you remembered how to put together the general solution via the superposition principle and did very well in expanding the nonzero boundary condition into a Fourier series. There were hardly any errors in the calculation of the Fourier coefficients, which I was very impressed with. Only occasionally a factor 2 was forgotten.

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