MA40198 lab 1: Urchin growth

Always save your R code in a text file, with comments.

Echinus affinis is a species of deep sea urchin. In a 1985 paper Gage and Tyler reported data on the growth of *E. affinis* collected from the Rockall Trough. The data can be read into R using

```
urchin <- read.table("http://www.maths.bath.ac.uk/~sw283/MA40198/urchin.vol.dat")
```

and consist of volume measurements and ages for 142 individuals (the urchins have annual growth rings).

> **Exercise 0**. Produce a sensible plot of the data, in order to get a feel for what they look like. If you don't know what a sea urchin looks like, take a look at the Urchin section at:
> `http://www.marlin.ac.uk/sah/species_index.php?phy=Echinodermata`

Nisbet and Gurney (1998) suggested simple energy budget based arguments to arrive at a model for average volume as a function of age which is

$$\frac{\mathrm{d}V}{\mathrm{d}a} = \begin{cases} \gamma V & V < \phi/\gamma \\ \phi & \text{otherwise} \end{cases}$$

Where γ and ϕ are parameters. The initial volume is V_R, also a model parameter. Growth is in two phases — in the first the animal grows as fast as it can, given the food it can obtain, and in the second it grows less quickly, putting the surplus food energy into reproduction. The age at onset of reproduction is therefore

$$a_m = \frac{1}{\gamma} \log\left(\frac{\phi}{\gamma V_R}\right)$$

and the model can be solved analytically:

$$V(a) = \begin{cases} V_R \exp(\gamma a) & a < a_m \\ \phi/\gamma + \phi(a - a_m) & \text{otherwise} \end{cases}$$

> **Exercise 1**. Write an R function which has two arguments `theta` and `age`. `theta` is a vector containing values for V_R, ϕ and γ. `age` is a vector of ages at which to evaluate the urchin volume, according to the model. The function should return a vector of volumes corresponding to the supplied ages. Test that is seems to be working properly.

To turn the model into a statistical model we need to specify how the observed volume is related to the predicted volume. Some experimentation suggests that

$$\sqrt{V_{obs}} = \sqrt{V_{pred}} + \epsilon$$

where $\epsilon \sim N(0, \sigma^2)$ (and the ϵs for different observations are independent) is a reasonable model.

> **Exercise 2**. With pencil and paper, write down the log-likelihood of the parameter vector $\theta^{\mathsf{T}} = (V_R, \phi, \gamma, \sigma)$, given that you can produce a vector of predicted volumes, given any particular θ. Now write an R function to evaluate this log likelihood. Again `theta` should be the first argument and `age` the second, but this time there should also be a `vol` argument, which will contain the observed volumes. Hint: your log-likelihood function should call the function that you wrote for exercise 1. Test that the log-likelihood is larger for silly θ values, than for more sensible ones.

If you have time try to find a reasonable value for θ and then plot transects through the log-likelihood in which you vary one parameter, while keeping the others fixed. The idea here is to try to get a feel for the shape of the function.

Save your log likelihood function: you will need to use it in the next lab.