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MA10193: Mathematics 2

H1. Write out the matrix in full, where its elements are given by

a) $a_{ij} = 2i + j + 1, \;\; 1 \leq i, j \leq 3,$ [1]

b) $a_{ij} = (j-1)^i, \;\; 1 \leq i \leq 2, 1 \leq j \leq 4.$ [1]

H2. For which values of $x, y \in \mathbb{R}$ is

a)
$$\begin{bmatrix} 2 & 2x+1 \\ y-2 & x \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 0 & y \end{bmatrix},$$
[2]

b)
$$\begin{bmatrix} -5 & y \\ x+1 & -2 \end{bmatrix} = \begin{bmatrix} -5 & 3x \\ 2 & 7 \end{bmatrix}?$$
[2]

H3. Given that
$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 2 & -1 \\ 0 & 3 & -1 \end{bmatrix},$$
find the matrices $A + B$, $A - B$ and $4A - 2B$. [3]

H4. Let $A = \begin{bmatrix} 2 & 0 & -4 \\ -7 & 0 & 5 \\ -4 & 3 & 1 \end{bmatrix}$. Find a matrix B such that $A + B$ is the identity matrix I_3. [2]

H5. Given that
$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 1 & -2 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 0 \\ -1 & -2 \\ 1 & 2 \end{bmatrix},$$
verify the distributive law $A(B + C) = AB + AC$ for the three matrices. Can you calculate CB? Can you calculate $C - B$? [4]

H6. Let
$$A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \\ 0 & -1 \\ -2 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$
Say which of the following expressions are defined, and calculate those that are:

a) A^2, B^2, C^2. [3]

b) $AA^{\mathrm{T}}, B^{\mathrm{T}}B, CC^{\mathrm{T}}, C^{\mathrm{T}}C$. [4]

c) $C^{\mathrm{T}}AB$, $(AB)^{\mathrm{T}}C$. [2]

H7. If

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -4 & 0 \\ 2 & 2 & -1 \end{bmatrix},$$

show that $(AB)^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}}$. [3]

H8. If $\underline{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$, $\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & -1 \\ 8 & -3 \end{bmatrix}$, find $\underline{x}\,A\,\underline{y}$ and $\underline{y}\,A\,\underline{x}$. [2]

H9. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ b & c & 1 \end{bmatrix}.$$

Find A^2. For which relation between a, b and c is $A^2 = \mathrm{I}_3$? [3]

H10. Find all matrices $A = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ that commute with $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. [4]

Please hand in solutions by 11:15am on Thursday 13 February, in the pigeon hole marked MA0193 in 4W on level 1.

Please note in your solutions if you have difficulties with a topic or homework problem and would like a particular discussion in the tutorial.