

— H12 We follow the arguments for Laplace's equation

Step 1: Assume first  $u_t - \Delta u < 0$  in  $U \times (0, T)$ .

• If  $u$  attains its maximum in  $U \times (0, T)$

We consider the then  $u_t = 0$  and  $\frac{\partial u}{\partial x_j} < 0$

quantities which  $\Rightarrow u_t - \Delta u \geq 0$ , a contradiction

appear in the heat

equation and • If  $u$  attains maximum on  $U \times \{T\}$

try to determine (i.e.,  $u(x, T) = \max$  for some  $x \in U$ ),

their sign. Then necessarily  $u_t(x, T) \geq 0$ , since otherwise

$u(x, t)$  would be greater than  $u(x, T)$  for  $t$  close to  $T$ .

$\Rightarrow u_t - \Delta u \geq 0$ , again a contradiction

(the argumentation for  $\Delta u$  remains unchanged)

$\Rightarrow$  maximum must be attained on  $U$  for  $t=0$

or on  $\partial U$  for  $0 \leq t < T$

If  $u$  is a temperature, this implies the temperature in the body can not exceed the temperature provided at  $t=0$  or at the boundary of the body during the time evolution